Reliability requirements for structural engineering decision making

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Synopsis:
The specification of reliability requirements is addressed in this discussion note. The document is in a draft stage and comments to the author are not only welcome but encouraged.

1 Introduction

Structures should be designed, operated, maintained and rebuild such as to support societal functionality and enhance sustainable societal development during their service life [1]. The decisions in relation to this are herein referred to as Structural Engineering Decisions. These decisions have to be made in relation to the societal preferences and to the anticipated effect of the decisions. Normative decision theory, as introduced by von Neumann and Morgenstern already in 1943, [2] provides the general framework for engineering decision making and provides the decision maker with a rational basis to act. Accordingly, the decision maker does evaluate and rank the utility of different decision outcomes in terms of expectations. The evaluation of expected utilities for structural engineering decision problems is generally associated with large aleatory and epistemic uncertainties that have to be considered. A consistent framework that accounts for information and uncertainty in decision making is Bayesian Decision Analysis as presented in [3].
It is here claimed that Bayesian Decision Theory represents the basis that allows for the rational foundation for engineering decisions and provides the means for analysing whether the decision is selected on the best available basis. However, many practical situations do not allow for such a formal treatment and more operational and simplified decision rules are applied in practical engineering decision making.
Decision making concerning design and assessment of structures is addressed in ISO 2394, [1]. Here, three different levels of detail of decision analysis are distinguished; risk-informed decision making, reliability based design, and semi-probabilistic design, see Figure 1.

1.1 Risk informed decision making

Risk informed decision making follows broadly the scheme of Bayesian Decision Analysis as outlined in the introduction. The analysis is performed in the confines of a specified model universe. The model universe is chosen such that it represents the decision problem, i.e. the effect of different decision alternatives, on the preferences with sufficient

	Commonly applied when:	Objective:
Risk-informed decision making: - decisions are taken with due consideration of the decision makers preferences.	Exceptional design situations in regard to uncertainties and consequences.	Maximize the expected utility of the decision maker.
Reliability-based design and assessment: - estimation of the probability of adverse events.	Unusual design situations in regard to uncertainties.	Satisfy reliability requirements.
Semi-probabilistic: - safety format prescribing design criteria in terms of the design equations and the analysis procedures to be used.	Usual design situations in regard to consequences and uncertainties. Default method of most design codes.	Satisfy deterministic design criteria.

Figure 1: Levels of Structural Engineering Decision Making according to [1].

level of detail. In the structural engineering context, decision alternatives may include structural measures, i.e. the dimension of cross sections, the choice of material grades, the design of a superstructure, repair and strengthening etc., and non-structural measures as e.g. the implementation of quality control and checking, inspection or the installation of structural health monitoring. Simplification or enhancement in the representation of the decision problem could also be considered as a non-structural measure. It is assumed that executive decision making is not mechanically following the results of the formal decision analysis but that the well documented assessment informs the decision maker, such that he or she is able to identify his or her final decision.

Risk informed decision making is very flexible and can be applied for systems of different scale in space and time. Depending on the definition of the system boundaries, structural performance attributes that are related to societal preferences as e.g. sustainability, resilience, robustness and reliability can be addressed within an analysis. The risk informed decision framework can also be applied for the calibration of lower level decision making methods as will be discussed later in this document. Guidance and standardisation for risk based decision making can be found in ISO2394 [1]. However, the flexibility and generality of the method requires a large amount of expertise and experience from the person or group of persons elaborating on a risk informed decision analysis.

It is important to note that "risk", in the context of risk informed decision making, is only utilised as a comparative measure and should not misunderstood as an absolute attribute of the physical structure or structural system. It remains an attribute to the analysis (based on the corresponding assumptions, simplifications and associated to epistemic uncertainties) and should be always seen and communicated conditional to that.

1.2 Reliability based design

In reliability based design, the design decision is chosen such that it complies to a predefined reliability requirement, i.e. the reliability requirement is considered as a criterion for simplified decision making. It is possible to assess the effect of a decision on the failure probability on a component level, i.e. only one possible failure mode is considered, or on a system level, i.e. the interaction of different failure modes in a structure are considered.

However, as consequences are not represented explicitly in reliability based design, subordinate structural performance attributes as robustness, sustainability and resilience can not be considered explicitly.
Reliability based design is addressed in the international standard ISO2394 [1], more detailed guidance on structural reliability methods and uncertainty representation in regard to models, structural resistance and structural demands is found in the Probabilistic Model Code of the Joint Committee on Structural Safety, [4].
The specification of reliability requirements for reliability based design is addressed later in this note.

1.3 Semi-probabilistic design

The semi-probabilistic approach corresponds to the lowest level of detail. Here, a design decision is chosen such that it complies with the criterion that a design value of a resistance is larger than a design value of a corresponding load effect. Design values for the load bearing capacity R_d are chosen to have a sufficiently low non-exceedance probability and design values for loads E_d are chosen to have a sufficiently low exceedance probability such that the design criterion in the limit ($R_d = E_d$) corresponds to the required level of reliability.
In the so-called load and resistance factor design (LRFD) format [5] design values are estimated based on characteristic values and partial safety factors γ as e.g. $R_d = R_k/\gamma_M$ for resistance variables and $E_d = \gamma_E E_k$ for the effects of applied loads. Both, the definition of the characteristic value and the choice of partial safety factor, is made in order to meet the reliability requirements. However, the correspondence to reliability requirements is generally made for domains of design situations. For the representation of these design situations generalised assumptions in regard to consequences and uncertainties are made. With semi-probabilistic design structural reliability on a component/failure mode level can be controlled. The explicit consideration of the interaction of failure modes in a structure, i.e. system effects, is not accommodated.
The principles of semi-probabilistic design are outlined in [1]. It is the method of choice for most structural design decision problems and executive guidance and standardisation is found in several national and international design standards as, e.g. the Eurocodes [6]. The specification of reliability requirements for semi-probabilistic design is further discussed later in this note.

1.4 Relevance and correspondence

The different levels of engineering decision making are all relevant and support the superordinate objective of the safe and optimal development and maintenance of structures in the build environment. And the approaches on the different levels closely correspond to each other. The ability to account for the particular conditions of specific design situations and therefore identify a more optimal design solution is increasing with increasing level. That is the reason why higher level approaches are used to verify or calibrate lower level approaches. However, the complexity and difficulty is increasing with increasing level and from a regulative perspective, i.e. where a unified set of rules and assumptions for broad application shall be standardised, semi-probabilistic approaches are advantageous since their inherent low level of detail goes along with the allowance for broad generalisation.
The design, calibration and layout of simplified decision making approaches, has to be based, directly or indirectly, on risk informed decision making, as this is the only level of detail that allows for the explicit consideration of the superordinate objectives in engi-

neering decision making.

2 Requirements for reliability based design

The specification of requirements for reliability based design is basically seen as a calibration process and the corresponding formal calibration process should be followed [7], i.e..

- *Scope:* Define the class of structures to which the reliability requirement is intended to apply.
- *Adverse states:* What kinds of adverse states are addressed by the reliability requirement. This generally refers to failure modes that a represented by limit states.
- *Format:* A format for reliability based design is $\beta^{(\tau)}(\mathbf{p}) \leq \beta^{(\tau)}_{target}$, where $\beta^{(\tau)}(\mathbf{p})$ is the reliability index corresponding to the reference period τ attained by the structural design $\mathbf{p}$. $\beta^{(\tau)}_{target}$ is the reliability requirement corresponding to the same reference period τ.

Compared to risk informed decision making reliability based design represents a simplification and usually the same reliability requirement is used for a domain of (design) decision situations. The calibration of a requirement for reliability based design is seen as a risk based decision problem. The question to answer is: What choice of reliability requirement does, when applied in reliability based design, maximise the expected utility for the decision maker?

2.1 Set up of the risk based decision problem

In order to set up the formal decision problem and in order to follow the above mention systematic approach to code calibration, the following needs to be further specified:

- Decision maker: If the reliability requirement is calibrated in a regulatory context, i.e. to be implemented as a requirement in standardisation, the decision maker corresponds to the society in which the standard is applied. Code committees act as a representative of the society and should aim to represent the preferences of the society in the decision making. In some cases, the reliability requirement may be intended to be used in the context of a private endeavour. Then the decision maker has to be specified accordingly. However, if safety of personnel or the environment is of concern, it has to be justified that the preferences of the society are not impaired by the decision.
- The domain of decision situations considered: The characteristics of structural design and reassessment decision problems are rather varied. Dependent on the chosen system boundaries, events of interest might span from temporary or permanent loss serviceability, the realisation of one single failure mode, the development of a cascading realisation of several failure modes towards the collapse of large spatially distributed structures, etc. Furthermore, structures can support a large variety of societal activities, from storing straw towards producing nuclear power and consequently events of interest (in reliability analysis broadly referred to as failure) can imply a large variety of consequences. Available decision alternatives or measures to

increase reliability might have different cost efficiency (i.e. cheaper or more expensive material, smaller or larger uncertainty). Discrete domains of decision situations have to be defined and the decision problem has to be formulated such that the attributes of the entire domain are represented. The broader and more general the domains are chosen, the more accuracy (compared to risk informed decision making) is lost - on the other hand, the more general the reliability requirement can be applied.

- The reliability requirement is indented as a criterion for decisions on structures; in a sense it can be seen as a decision rule that is continuously applied in many design and reassessment situations distributed in time and space. For the calibration of the reliability requirement it is thus necessary to represent rather the decision situations for which the requirement is used than the physical structures.

The purpose of structures is to support societal activities over time. When developing simplified decision rules on structures, as e.g. the reliability requirement for reliability based design, the need for supporting societal activity should be considered. This can be considered with the continuous renewal assumption [8]. Note that renewal rather refers to the re-occurrence of a decision problem with similar attributes and for which the same decision rule should apply. Rüdiger Rackwitz formulated a risk based decision problem that allows for the required generalisations [9]. Accordingly the risk optimal design p^* is found by the following minimisation problem:

$$p^* = \arg\min_{p} E\left[C_{tot}\left(p\right)\right] \tag{1}$$

i.e. the utility representing the preferences of the decision maker is expressed by the expected costs C_{tot}. p^* is representing the decision parameter that is chosen in the design and that has an effect on the probability of failure. It is clear that in a practical design situation many different parameters might be chosen simultaneously. Here, however, it is assumed that it is only one parameter.
The total present expected costs are defined as [9]:

$$\begin{aligned} E\left[C_{tot}\left(p\right)\right] &= C_{constr}\left(p\right) + E\left[C_f\left(p\right)\right]\frac{1}{\gamma} + E\left[C_{obs}\left(p\right)\right]\frac{1}{\gamma} \\ &= \left[C_0 + C_I p\right] + \left[C_0 + C_I p + H\right]\frac{\lambda P_f^{(1a)}\left(p\right)}{\gamma} + \left[C_0 + C_I p + D\right]\frac{\omega}{\gamma} \end{aligned} \tag{2}$$

Equation (2) consist of the following:

- Construction costs: $C_{const}(p) = C_0 + C_I p$ are the construction cost with a part $C_I p$ that is proportional to p, and a part C_0 that is independent of p.

- Expected annual failure cost: $E\left[C_f(p)\right] = \left[C_{const}(p) + H\right]\lambda P_f^{(1a)}\left(p\right)$, here H is introduced as a nonstructural failure cost, the construction costs are part of the failure cost as a similar decision (similar investment into p) is expected after failure. The failure cost are considered as annual expectations, so they are multiplied by the annual failure probability $P_f^{(1a)}\left(p\right)$. λ is introduced as the rate of a Poisson process with $\lambda = 1$ in order to interpret the annual failure probability as a rate of occurrence.

- γ is the annual interest rate that is selected as the societal interest rate $\gamma = \gamma_S$ if the preferences of the society are represented, or as the private interest rate $\gamma = \gamma_E$, if the optimisation is done relative to entrepreneurial preferences.

- Expected obsolescence cost: $E\left[C_{obs}\right] = \left[C_{const}(p) + D\right]\omega$, here D represents the demolition cost and ω is representing the expected rate at which the decision on p becomes obsolete.

The design parameter $p = p^*$ that is minimising the expected total cost is found by

$$\frac{d}{dp}\left\{C_0 + C_I p + \left[C_0 + C_I p + H\right]\frac{\lambda P_f^{(1a)}(p)}{\gamma} + \left[C_0 + C_I p + D\right]\frac{\omega}{\gamma}\right\}\Bigg|_{p=p^*} \equiv 0$$
$$\Rightarrow \frac{C_0 + C_I p^* + H}{C_I} = \frac{1 + P_f^{(1a)}(p^*)\frac{1}{\gamma} + \frac{\omega}{\gamma}}{-\left.\frac{dP_f^{(1a)}(p)}{dp}\right|_{p=p^*}\frac{1}{\gamma}} \tag{3}$$

A simplified approximation can be formulated by considering that for typical structural engineering decision situations it is $P_f(p^*) \ll \omega + \gamma$:

$$\frac{C_I \cdot (\gamma + \omega)}{C_0 + C_I p^* + H} \approx -\left.\frac{dP_f^{(1a)}(p^*)}{dp}\right|_{p=p^*} \tag{4}$$

Note that the expression for the ratio between the marginal safety costs (C_I) and the total failure costs ($C_0 + C_I p^* + H$) can be further simplified. Usually, $C_I p^* \ll C_0$ since the construction costs are dominated by the fixed costs. In these cases, the ratio on the left side of 4 is approximately equal to $C_I / (C_0 + H)$.

$$\frac{C_I \cdot (\gamma + \omega)}{C_0 + H} \approx -\left.\frac{dP_f^{(1a)}(p^*)}{dp}\right|_{p=p^*} \tag{5}$$

The optimal design p^* can be estimated from Eq. (4) and (5) when the functional relationship $P_f^{(1a)}(p)$ is known analytically in the vicinity of $p = p^*$. In other cases, the equation must be solved numerically. It is interesting to note that the optimal choice of p does only depend on $dP_f^{(1a)}(p)/dp$ and not on the absolute values of $P_f^{(1a)}(p)$, i.e. in the optimisation subject to p it is only necessary to represent the gradual change of $P_f^{(1a)}$ by changing p. This makes the results of the optimisation insensitive against the non-inclusion of failure scenarios those probability of occurrence cannot be influenced by gradually changing p. An example is the potential presence of gross human error. This definitely has a strong effect on the estimated absolute probability of failure, however, if it is assumed that the probability of failure conditional on gross human error is not (or very weakly) influenced by the particular choice of p, the optimum choice of $p = p^*$ derived by neglecting gross human error is still valid.
A reliability requirement can be determined from the above optimisation, i.e. if the choice of $p = p^*$ is minimising the expected total costs, then $P_f^{(1a)}(p^*)$ and $\beta(p^*) = -\Phi^{-1}(P_f^{(1a)}(p^*))$ are the corresponding failure probability and reliability index of the optimal choice. If for a decision problem at hand, it can be assumed that the attributes of the decision problem are sufficiently similar to the assumed attributes contained in Eq. (4), this probability / reliability should be chosen as a target. However, since such a target is expressed in terms of an absolute value it is of high importance that the reliability target is only applied to sufficiently similar scenarios.

2.2 Risk acceptance criteria

The economic optimisation that is obtained from Eq. (4) ensures that resources are expended optimally from a financial point of view. But it does not guarantee that the optimum decision $p = p^*$ is consistent with the societal preferences in regard to life safety. I.e. if human lives are at risks it has to be ensured that societal resources are allocated efficiently for preventing possible fatalities associated with structural failure. The marginal life-saving costs principle can be applied to derive risk acceptance limits [1, 10]. The principle ensures that the societal resources are allocated to efficient risk-reducing measures, i.e. measures that can save one additional life at a cost that the society is willing (or able) to pay. This is here referred to as the Societal Willingness To Pay (SWTP). The SWTP is multiplied with the expected number of fatalities given structural failure and inserted to the objective function in Eq. (2) which leads to the specification of the acceptable domain in Eq. (6), [1, 11].

$$-\frac{dP_f^{(1a)}(p)}{dp} \leq \frac{C_I(\gamma_S + \omega)}{SWTP \cdot N_F} = K_1 \tag{6}$$

The constant K_1 is introduced in [1] as an indicator. Here, also typical values of K_1 are given. The minimum acceptable design (p_{acc}) just satisfies the inequality in Eq. (6) and depends on:

1. the uncertainty involved in the problem through the term $dP_f^{(1a)}(p) \big/ dp$,
2. the marginal safety costs C_I measured in monetary units,
3. the societal ability to pay for saving one statistical life $SWTP$, measured in monetary units,
4. the number of expected fatalities given structural failure N_F, and
5. the obsolescence rate ω and the societal interest rate γ_S.

The condition for which the optimal design is within the acceptable domain (i.e. $p^* \geq p_{acc}$) is obtained inserting Eq. (6) into Eq. (5) :

$$\frac{C_I \cdot (\gamma_S + \omega)}{C_0 + H} \leq K_1 \tag{7}$$

How the SWTP can be quantified is discussed in section 2.3.5.

2.3 Selection of parameters for calibration

The purpose of the calibration exercise is to derive reliability requirements for reliability based design decisions, such that the results correspond approximately to the optimal decision. This is possible based on Eq. (5) if the parameters represent the corresponding class of decision situations. For the quantification of these parameters the following information is considered relevant.

2.3.1 The functional relationship between $P_f^{(1a)}$ and p

The functional relationship between $P_f^{(1a)}$ and p in Eq. (4), (5) and (6) has to be formulated such that it represents well all structural decision situations for which the reliability requirement is supposed to be valid. This appears to be a challenging task, as "failure" might relate to the realisation of a single failure mode or system failure, i.e. the realisation of a combination of single failure modes leading to structural failure in a system. Correspondingly, the mathematical representation of "failure" might look quite different, i.e.

the linear- or non-linear combination of relevant variables representing loads and resistances that are quite different in regard to magnitude and scatter for different cases. The representation of all these different possible functional relationships by a single expression calls for significant generalisation, and that can only be provided by drastic simplification. In [9] a simple limit state function containing one resistance variable R and one load effect variable S is suggested to represent failure. The decision variable p is introduced as a kind of central safety factor that is linearly scaling R. The limit state function reads $g(r,s) = pr - s$ and failure is defined as $\mathcal{F} = g(r,s) < 0$. (Note that capital letters are used to indicate random variables; small letters are used to indicate realisations of random variables). It is sufficient to represent resistance R and load effect S with unitary location parameter, e.g. with mean value equal to one, such that the variables can simply be characterised by distribution type and coefficient of variation. This represents four degrees of freedom and it is surprising how flexible this simplified model can represent, at least in the vicinity of the optimal decision $p = p^*$, the functional relationship between $P_f^{(1a)}$ and p of specific reliability problems that are much more complex. The problem is to identify a combination of distribution types and coefficients of variation that represent a class of structural decision problems.

2.3.2 Consequences H and costs C_0 and C_I

The consequences are highly variable among different decision situations and furthermore they are hard to estimate for specific problems and therfore associated to large uncertainties. However, for the identification of the optimum decision $p = p^*$ and the corresponding failure probability $P_f^{(1a)}(p^*)$ it is sufficient to represent the consequences with their expected values, i.e. their distribution and scatter are irrelevant for the identification of the optimal design.

The expected construction costs.
The construction cost term $C_{constr}(p)$ includes all the costs that are re-occurring any time the decision is re-implemented after an obsolescence or failure event. In detail, $C_{constr}(p)$ includes:

- The costs of non-structural parts, installation, design, assembly and so on denoted by C_{NST}. These costs are independent of p (in the vicinity of the optimum). For example, the costs of designing a structural element are included in C_{NST} since it does not change with small variations of the element cross-section.

- The costs of structural parts $C_{ST}(p)$ that depend on the design parameter (e.g. the cost of the material utilised). $C_{ST}(p)$ is linearised in the vicinity of the optimum design as $C_{ST}(p) \approx C_{ST}(p^*) + C_I(p - p^*)$. Since the optimum design is not known before optimisation and only a general approximation of the construction costs is sought, the linear approximation can be performed at $p = p_0$, where p_0 is the design corresponding to a chosen reliability index close to the anticipated optimum that is, usually, between 3 and 5 for ultimate limit states. In more general situations, the safety of a structure might be increased by changing several design variables. In this case, the argument of the optimisation (p) should be the most effective of them, i.e. the design parameter with lowest C_I.

In summary, the construction costs are expressed in Eq. (8). This linear relation is only valid in the vicinity of the optimum.

$$C_{constr}(p) = C_{NST} + C_{ST}(p) \approx C_{NST} + C_{ST}(p_0) + C_I(p - p_0) = C_0 + C_I p \quad (8)$$

It is worth highlighting that the optimal reliability level is not depending on the demolition costs D, see Eq. (2). This is a consequence of the fact that the demolition and clearance costs are considered independent of the decision variable p, which is a reasonable assumption for most cases.

The expected failure costs H.
The costs associated with failure H include the monetary value of all consequences beyond the cost of re-implementing the decision, C_{constr} such as the damage to property, cost for interruption of the societal activity (e.g. loss of production) and also compensation costs for damage to the environment or injuries/fatalities.

The estimation of the tangible and intangible costs included in H is not a trivial task. A possible approach consists in estimating the implicit value of H from the existing best practice, i.e. the existing design codes. With the framework proposed in [12], H that is implicitly accounted for in the existing design practice is estimated under the assumption that the existing practice is optimal in average. This hypothesis seems strong but at the same time reasonable. In fact, the safety level of the current design code seems not to be so high to impede the construction of the necessary structures, and, at the same time, the society appears to accept the current risk to life associated with the use of the built structures. When performing this estimation, it is important to select a class of structures similar to the one that are subject to optimisation.

The characteristics of the failure mechanisms should be considered when evaluating the expected number of fatalities and other indirect consequences given failure. Brittle failure mechanisms and non-redundant structural systems might cause more fatalities and larger consequences compared with ductile failure mechanisms that warn the occupants before the collapse. This might be represented with higher or lower values for H/C_0 correspondingly.

2.3.3 The obsolescence rate ω

The obsolescence rate ω corresponds to the expected rate a structural engineering decision becomes ineffective and is reconsidered, i.e. the decision becomes obsolete and the rate is often implemented as the reciprocal expected service life of a structure, $1/T_{life}$. The service life T_{life} is given in the design standards and depends on the type of structure. Conventionally, normal structures have an expected life of 50 years [6]. In the context of decision making, it is sufficient to represent the obsolescence rate as an expected value.

2.3.4 The long term annual real interest rates γ_S and γ_E

The annual interest rate for the optimisation is chosen by the decision maker performing the optimisation (a societal decision maker is choosing γ_S). On the opposite, the risk acceptance should always be assessed with the societal interest rate γ_S both by the private and the societal decision makers. γ_S is derived considering the needs of future generations, while γ_E does not necessarily consider them. The societal interest rate lies approximately between 2 % and 5 % [11]. Typically, γ_E for a private decision maker is larger than γ_S. The quantification of interest rates that well represent the future economical and societal developments is associated with large uncertainties. However, in the context of decision making, it is sufficient to represent interest rates as expected values.

2.3.5 The societal willingness to pay for saving one additional life $SWTP$

The $SWTP$ corresponds to the amount of money that should be invested into saving one additional life for a given life risk reduction activity. The $SWTP$ is expressed in monetary terms and can be derived e.g. by means of the Life Quality Index (LQI) [13]. The LQI index informs about the revealed preferences of the society for investments into life safety by estimating the ability of a national economy to allocate monetary resources in risk reducing activities.

Values of the $SWTP$ that are derived from the LQI index are shown in Table 1 for some selected countries, a more complete Table is given in [1]. The values in the Table are provided for different societal interest rates γ_S for considering the discounting of the future costs and benefits of the risk-reducing measures.

Table 1: SWTP for five selected countries from [1] for γ_S equal to 2 %, 3 % and 4 % (all numbers in thousands purchasing power parity US dollars).

Country	2008 GDP	SWTP		
		γ_S =2%	γ_S =3%	γ_S =4%
Australia	35 624	4 840	4 298	3 843
Brazil	9 517	804	712	634
Norway	49 416	3 937	3 500	3 129
Mali	1 043	54	48	43
US	42 809	3 187	2 833	2 543

The use of monetary units in the context of risk acceptance should not be misunderstood. Indicators such as the cost of a statistical life or the money the society is willing (or better "able") to pay for saving one additional life ($SWTP$) are not to be considered the monetary value of a individual person's life, which has of course a value that cannot be expressed in monetary terms. The mentioned indicators are derived from small marginal changes in the probability of having a fatality caused by failure.

2.3.6 Expected values

The cost terms H, C_0, C_I and N_F but also ω and γ_S are in general uncertain. A similar failure event might lead to different consequences depending, for example, on the occupancy of the building. All these terms need to be evaluated up to their expected value only since they affect the expected total cost linearly (compare Equation (2)). If in addition it can be assumed that they are independent of each other their probability density functions are irrelevant for the optimisation, see [3]. It follows that the values for the optimisation are not the highest or extreme values associated with the corresponding events, but the expected ones. However, the expectation operator is omitted in the text in order to ease the notation.

When evaluating the expected values of H and N_F, attention should be paid in cases where the assets and the lives lost in the event of failure might be themselves a significant cause of failure. A representative example might be the failure of a grandstand in a stadium due to static or dynamic load induced by the occupants. In this case, the likelihood of failure increases with the occupancy. This effect should be considered both in the formulation of the objective function and in the representation of the variables.

2.4 Graphical representation of requirements for reliability based design

Eqs. (4) and (6) can be used to calibrate reliability requirements for classes of structures and represent these in tabulated form. Such a table for reliability requirements derived based on the optimally criterion (Eq. (4)) can be found in the Probabilistic Model Code of the Joint Committee on Structural Safety [4]. Here, classes of structures are defined in regard to the consequences of failure and the cost of safety measure both relative to the fixed construction cost. The relative consequences ρ are defined as $\rho = 1 + H/C_0$ and three classes are introduced; "minor": $\rho < 2$, "moderate": $\rho = [2, 5)$ and "large": $\rho = [5, 10)$. The relative costs of safety measure are introduced qualitatively as three classes referring to "large", "normal" and "small". The relative cost of safety measure can be related to C_I/C_0 but they are also related to the the uncertainty of the reliability, the obsolescence rate and the interest rate.
The same table is found in [1], where additionally a table of reliability limits derived based on the acceptance criterion (Eq. (6)) is given. The acceptable reliability is only dependent on the relative cost of safety measure.
In the absence of a clear definition of the relative costs of safety measure alongside the tables for reliability requirements the specification of a reliability requirement is difficult, and any selection for a specific value remains ambivalent and in many cases not consistent with level of detail and the methodological rigour that is aimed at in a reliability analysis.

Alternatively, a simple plot for deriving $\beta_{opt}^{(1a)} = \beta^{(1a)}(p^*)$ based on Eq. (4) and $\beta_{acc}^{(1a)} = \beta^{(1a)}(p_{acc})$ based on Eq. (6) is presented in [14]. The plot allows individuating the optimal and acceptable safety level for each case at hand accounting explicitly for the characteristics that influence the optimum and the acceptable domain. The plot is depicted in Figure 2 and each problem at hand is to be characterised with the following two indicators, I_{opt} and I_{acc}:

$$I_{opt} = log\left(\frac{[(1 + \frac{H}{C_0})}{[\frac{C_I}{C_0} \cdot (i + \omega)]}\right) \tag{9}$$

$$I_{acc} = log\left(\frac{SWTP \cdot N_F}{C_I\,(\gamma_S + \omega)}\right) = log\left(\frac{1}{K_I}\right) \tag{10}$$

The plot in Figure 2 represents uncertainty in the resistance variable and in the load variable using log-normal distributions and the simple limit state function $g(r, s) = pr - s$, compare section 2.3.1. More complex reliability problems can be represented by this simple relationship, as discussed above. However, the curves in Figure 2 are slightly different when, e.g. a Gumbel distribution is used for the representation of the load variable. This is demonstrated in [15], where also two examples are given on how to apply the graph in practical engineering decision problems.

3 Calibration of semi-probabilistic design codes

Systematic approaches to the calibration of semi-probabilistic design codes have been addressed e.g. in [16], [17], [12], [18], [19] and is standardised with special focus on the reliability based calibration of semi-probabilistic design codes in [1].

A semi-probabilistic design code consists of several so-called reliability elements $\mathbf{r}$, e.g. partial factors, load combination factors, characteristic values. These reliability elements

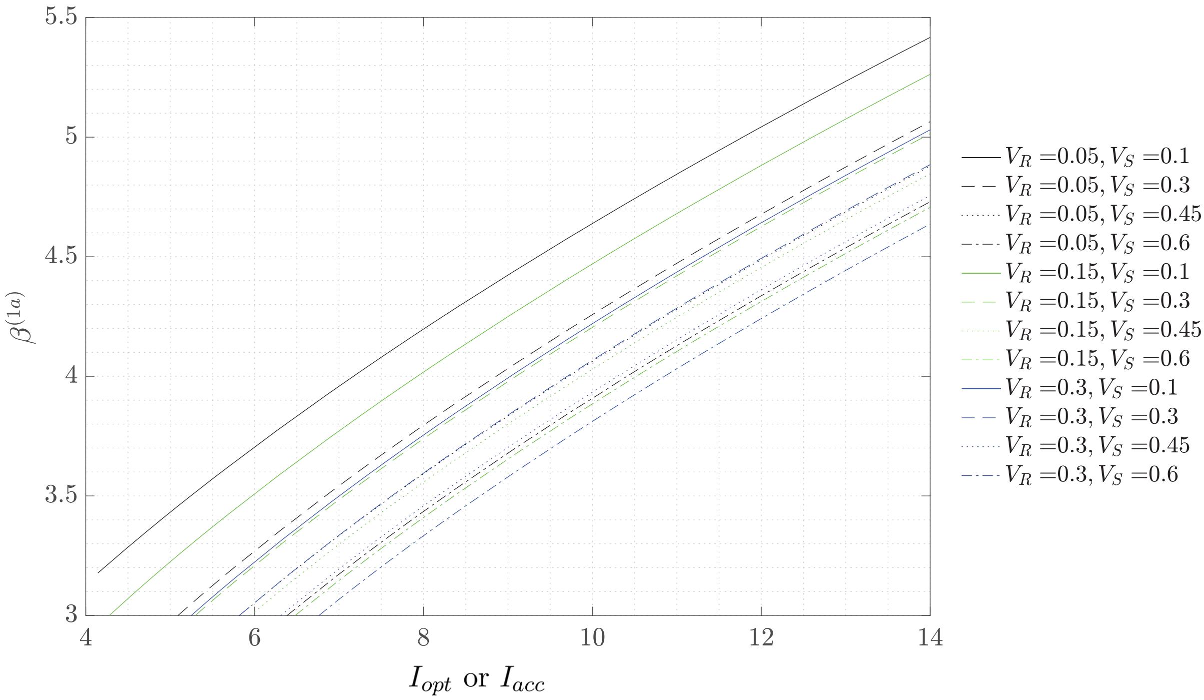

Figure 2: Plot for estimating the optimal and the minimum acceptable reliability indices, based on I_{opt} (Eq.(9)) and I_{acc} (Eq.(10)) correspondingly. R and S are represented as log-normal distributed random variables with the corresponding coefficients of variation V_R and V_S.

are partly chosen and partly calibrated such that the decisions that are based on the semi-probabilistic design code are consistent with the overall societal objective, i.e. the safe and efficient development and maintenance of structures. Traditionally, semi-probabilistic design codes are calibrated based on reliability and not based on risk. For reliability based code calibration the simplified objective is to provide consistent reliability levels among a class of design decision situations. This can e.g. obtained by minimising the squared difference between a reliability target β_t and the reliability reached for the different design situations i that are jointly considered in the calibration. The optimal set of reliability elements $\mathbf{r}$ is thus identified e.g. with

$$\mathbf{r}^* = \arg\min_{\mathbf{r}} \sum_i \left(\beta_i(\mathbf{r}) - \beta_t\right)^2 \tag{11}$$

An equation as Eq.(11) is also referred to as penalty function. Several different penalty functions for reliability based calibration of semi-probabilistic design codes have been formulated in literature, see [15] for an overview. There, it has also been shown that the particular definition of the penalty function has not a significant impact on the results of the calibration. A much more relevant assumption refers to the choice of the target reliability. Here, different reasoning is followed in the practical context of calibration:

- *Definition of β_t as a overall prescriptive requirement for structural reliability.* Here, a fixed requirement for structural reliability is introduced by, e.g. authorities, and this requirement is also used in calibration. Although, this is the most common interpretation of a reliability target, it relies on a misconception and a misinterpretation. Reliability, and similarly failure probability, is falsely considered as a property a structure can display. Reliability and failure probability, however, are only attributes of the (decision) analysis on structures and should only understood as such.

- *Defining β_t as reliability requirement derived from optimisation.* This can in principle be done following the Eq. (4) and using Figure 2 respectively. The design situations that are jointly considered in the calibration exercise would have been represented in terms of the uncertainty in the limit state and the consequences. However, so far this is practically not done in a strict manner. Instead, design situations that are jointly calibrated are considered as "regular" situations with "moderate" consequences of failure and "medium" relative cost of safety measure, according to [1] arriving on the recommendation for usual design situation which is $\beta = 4.2$ for a one year reference period. This definition of the requirement for reliability based code calibration is also not unproblematic. Design equations as they are presented in the semi-probabilistic design code have evolved over a long period of time and represent the long term accumulation of engineering experience and expertise. This is very good, but during this development uncertainties in the representation of physical phenomena by the corresponding design equations have not been considered explicitly but implicitly by the introduction of conservative assumptions leading to model bias. It is in general very difficult to identify and quantify this model bias which might be rather different in magnitude for different design equations. The biases, also referred to as "hidden safety", directly trigger the corresponding reliability of the design solutions and calibration of semi-probabilistic reliability elements to an absolute reliability requirement might not lead to the envisaged result.

- *Defining β_t as reliability that is represented by the design solution of a generally accepted design code.* If it can be stated that the reliability that is attained by implementing a design code is acceptable and also considered as sufficiently economic, the objective of the calibration exercises might reduce to the decrease of variation of reliability level among the design situations that are jointly considered. I.e. representative reliability level of the existing code (given the existing reliability elements) can be considered as the reliability target, and the minimisation of the penalty function is reducing the variability of the reliability. In [20], it was discussed how such a representative value for β can be identified. In a recent calibration study for the Eurocodes the average reliability level of the existing code (given the existing reliability elements) was considered as the reliability target [15]. The appeal of this relative calibration is that it is relative insensitive against modelling assumptions in regard to the uncertainty of the representation of variables. The results are also insensitive against model biases as long as it can be assumed that the bias affects all reliability elements that are subject to the calibration in the same way. However, the explicit assessment of the absolute safety level is not possible when only a relative comparison is done.

All listed interpretations of targets for reliability based calibration of semi-probabilistic design codes are lacking consistency. The design situations that a regulated by a semi-probabilistic design code are very in-homogeneous in regard to their representation by design equations and the corresponding accounted uncertainties and the inherent model biases. The introduction of an absolute value of a target reliability seems therefore not to be a feasible solution.

The problems could possibly be overcome by introducing minimal expected cost as a criterion. A direct risk based calibration of the reliability elements of a semi-probabilistic design code can be formulated as follows:

$$\mathbf{r}^* = \arg\min_{\mathbf{r}} \sum_i E\left[C_{tot,i}\left(\mathbf{r}\right)\right] \tag{12}$$

which contains analogous to Eq.(11) a summation over all design situations i that are considered jointly in the calibration. $E\left[C_{tot,i}\left(\mathbf{r}\right)\right]$ can be defined as in Eq.(2) but with the attributes representing i and a formulation for the failure probability that is in correspondence with the semi-probabilistic code format under consideration.

$$\mathrm{E}[C_{tot,i}\left(\mathbf{r}\right)] = \\ \left[C_{0,i} + C_{I,i}\left(\mathbf{r}\right)\right] + \left[C_{0,i} + C_{I,i}\left(\mathbf{r}\right) + H_i\right] \frac{\lambda P_f^{(1a)},i(\mathbf{r})}{\gamma} + \left[C_{0,i} + C_{I,i}\left(\mathbf{r}\right) + D_i\right] \frac{\omega_i}{\gamma} \tag{13}$$

The solution of Eq. (12) $\mathbf{r} = \mathbf{r}^*$ might be found numerically. Risk based calibration of semi-probabilistic design standards is not only more consistent, but it also allows to account for the variation/uncertainties on the structures that will be designed with a given calibrated design code. In fact, in reliability based calibration a set of representative structures is chosen arbitrarily and the uncertainty on this selection is not taken into account in the calibration. It can be expected that the identification of an absolute minimum is not straightforward to obtain due to the possible dependence among the reliability elements.

4 Summary and conclusion

The derivation of reliability requirements for structural design has been discussed. It has been demonstrated how requirements for reliability based design can be related to formal structural optimisation and risk acceptance criteria.
The analysis of the optimisation problem showed the four most important factors that influence the optimal reliability levels:

- the ratio between total failure costs and marginal safety costs;
- the uncertainty involved in the problem;
- the obsolescence rate; and
- the discounting rate.

The acceptability of the monetary optimum is not always satisfied. The acceptance criterion can be derived by the Marginal Life-Saving Cost Principle that was here implemented using the Life Quality Index (LQI). It was demonstrated that the acceptable threshold depends on:

- the marginal safety costs;
- the societal willingness to pay for saving one additional life;
- the number of expected fatalities given failure; and
- the obsolescence and the societal interest rates.

Reliability requirements in current standards keep only a few of these aspects into account for differentiating target and acceptable reliability. Therefore, plots for communicating target and acceptable reliability that allow accounting for all the mentioned aspects explicitly are suggested. Consequently, their use leads to higher reliability differentiation and thus, higher levels of structural optimally compared with the tables included in existing design standards. The input parameters were discussed with the aim of supporting practitioners in their selection.
The calibration of semi-probabilistic design codes have been discussed. Problems with consistency have been identified for reliability based calibration, as a definition of an absolute reliability target seems hardly reasonable. An alternative would be the direct risk based calibration of the reliability elements of a semi-probabilistic design code. The problem has been formulated and briefly discussed.

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APPENDIX

A Reliability requirements in the current Eurocodes (EN 1990:2002)

A.1 Overview

A basic requirement stated in the present version of the Eurocode is that structures shall sustain its anticipated loads with appropriate level of reliability and in an economical way (EN 1990:2002, 2.1(1)).
The term "appropriate level of reliability" is specified further in (EN 1990:2002, 2.2(3)) where its choice is related to relevant factors, including:

(a) the possible cause and /or mode of attaining a limit state;

(b) the possible consequences of failure in terms of risk to life, injury, potential economical losses;

(c) public aversion to failure;

(d) the expense and procedures necessary to reduce the risk of failure.

A formal differentiation according to (a), (c) and (d) is not further considered in the present version of the Eurocode.
According to (b), structures and components are classified, e.g. according the consequence of failure whereas the reliability levels might be given either for classified structures as a whole, or, for classified components (EN 1990:2002, 2.2(4)).
In EN 1990:2002, Annex B3.1, a classification of buildings and structures in terms of consequences is defined (3 classes). It is mentioned that importance of a failure mode for the consequences should be considered (EN 1990:2002, B3.1(2,3)). This might lead to different classification of different failure modes in one structure.
In EN 1990:2002, B3.2 **minimum** reliability levels for 3 consequence classes are recommended, see Figure .

The consequence classes have an implication on the partial factor design. In order to

Table B2 - Recommended minimum values for reliability index β (ultimate limit states)

Reliability Class	Minimum values for β	
	1 year reference period	50 years reference period
RC3	5,2	4,3
RC2	4,7	3,8
RC1	4,2	3,3

Figure 3: Extract from EN 1990:2002: Table B2.

reach different minimum reliability levels the (unfavourable) effects of loads are multiplied, or, the resistance is divided by a factor. For the load side the following factors are given:
Within the Eurocodes this concept is also referred to as "Reliability Differentiation".

Table B3 - K_{FI} factor for actions

K_{FI} factor for actions	Reliability class		
	RC1	RC2	RC3
K_{FI}	0,9	1,0	1,1

Figure 4: Extract from EN 1990:2002: Table B3.

A.2 The effect of "Reliability Differentiation"

A reliability differentiation for different consequence/reliability classes is implemented in the Eurocode by the application of the K_{FI} factors to the unfavourable load effects. The intention is to increase the reliability level for RC3 and decrease it for RC1 both relative to RC2.

Case study:

The quantitative effect of the application of the factors in Figure 4 on the reliability level is assessed in this case study. A generic design situation with one variable load and one permanent load is considered. Equations 6.10 a b from EN 1990:2002 are used as design equations. The following indicative representation of the basic variables is chosen: The

Table 2: Basic variables properties from and partial safety factors.

	Description		Distr.	CoV	Char. fractile	Partial safety factor (γ)
X_R	Model uncertainties	Struct. Steel	Lognormal	5 %	-	-
		Reinf. Concrete	Lognormal	15 %	-	-
		Glulam timber	Lognormal	10 %	-	-
R	Material strength	Struct. Steel	Lognormal	7 %	5 %	$\gamma_M = 1.00$
		Reinf. Concrete	Lognormal	17 %	5 %	$\gamma_M = 1.50$
		Glulam timber	Lognormal	15 %	5 %	$\gamma_M = 1.25$
G	Self-weight		Normal	10 %	50 %	$\gamma_G = 1.35$
Q	Yearly maxima variable load (e.g. wind)		Gumbel	40 %	98 %	$\gamma_Q = 1.5$

generalised limit state function is thus:

$$g(\mathbf{X}) = pX_RR - a_GG - (1 - a_G)Q \tag{14}$$

with $a_G \in [0, 1]$ is a variable representing the proportion between self-weight and variable load.

The design variable p is determined by the corresponding design equations 6.10a b (EN 1990:2002) as

$$p = max\begin{pmatrix} \frac{\gamma_M}{r_k}\left[a_GK_{FI}\gamma_Gg_k + (1 - a_G)K_{FI}\gamma_Q\psi_0q_k\right] \\ \frac{\gamma_M}{r_k}\left[a_G\xi K_{FI}\gamma_Gg_k + (1 - a_G)K_{FI}\gamma_Qq_k\right] \end{pmatrix} \tag{15}$$

In the current example the load combination factor is set to $\psi_0 = 0.6$, and to $\xi = 0.89$. The reliability indexes corresponding to a one year reference period for $K_{FI} = 0.90$ (red), $K_{FI} = 1.00$ (black), $K_{FI} = 1.10$ (blue) are displayed for different and materials in Figure 5.

When inspecting Figure 5, the following observations can be made:

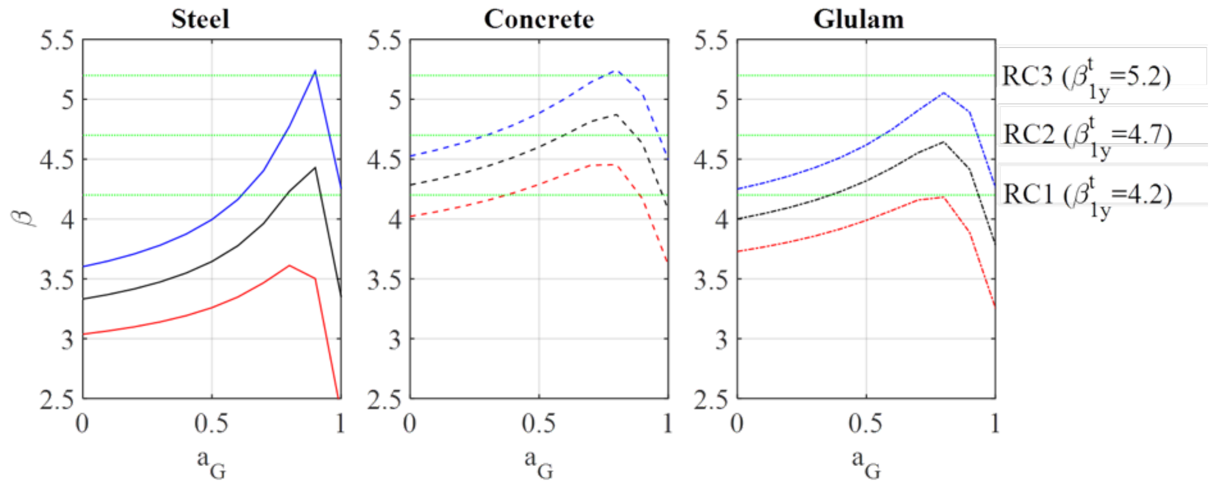

Figure 5: Yearly reliability for $K_{FI} = 0.90$ (red), $K_{FI} = 1.00$ (black), $K_{FI} = 1.10$ (blue). The green lines indicate the targets for the different reliability classes corresponding to a one year reference period.

- The differentiation of reliability level by applying $K_{FI} = 0.9, 1.0, 1.1$ is less than suggested by the targets for the reliability classes. (Note: the differentiation will be stronger if less variability on the load side is assumed and vice versa.)
- The variation of the reliability index for different design situations, i.e. different proportions of permanent and live load, is in the same order of magnitude that the full range of reliability class targets.
- With the assumed representation of the basic variables, the attained absolute reliability level is systematically too low compared to the minimum values given in Figure 3 (light green lines).

A.3 Eurocode reliability requirements for Code Calibration

In Annex C of EN 1990:2002 the calibration of partial factors on the basis of:

(a) similar design situations with "long experience for building tradition", and/or

(b) the recommended target reliability

is suggested, whereas it is mentioned that (a) is the leading principle for the existing partial factors.
For (b) the principles are outlined and it is referred to the reliability requirements as given in Annex B (Figure 3 in this document). It is noted that in Annex B the given reliability requirements are referred to as **minimum** values, in Annex C the same requirements are referred to as "target" values, i.e. as values that have to be attained **in average**. This difference has a strong implication on how the requirements are used as a reference in calibration, i.e. as absolute minimum values (not to be under-run in any design situation) or as target value for the average reliability level of many design situations.
If the reliability targets represent a reliability level of a common engineering structure that is thus broadly implicitly accepted by society, calibration according (a) or (b) lead in principle to the same result.

A.4 Discussion on the Eurocode reliability requirements

The target reliability requirement for consequence class 2 and a yearly reference period is $\beta = 4.7$.
It is observed that:

- This value is high if compared to the reliability of design solutions obtained by applying the existing Eurocode partial factor design concept (Figure 5).
- This value is not consistent with existing attempts to derive target and minimum reliability requirements based on optimization and based on life-safety considerations.
- The value is also high if compared to the reliability level international pre-Eurocode design codes. This comparison was made by IABSE WC1 and the results are presented in Figure 6 [Reference?].
- So far no documentation can be identified on how is derived and can be justified.

	3.1					3.5					4.0					4.5					5.0	
Argentina	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Canada	X	X	X	X	X	X	X	X	X	X	X	X	X	X								
China	X	X	X	X	X	X	X	X	X	X	X											
Denmark							X	X	X	X	X	X	X	X	X	X	X					
Estonia												X										
Germany												X										
Holland						X	X	X	X	X	X											
South Africa				X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X			
Spain												X										
Sweden							X	X	X	X	X	X	X	X	X	X	X	X				
UK																			X			
USA										X												

Figure 6: Target reliabilities β for a one year reference period as mentioned in a number of national and international codes (according to an inquiry by IABSE Working Commisison 1), adapted from [Reference?].

B On the choice of reference period for reliability targets

B.1 Background

For practical problems, probability of failure and the reliability index correspondingly is always conditional to a specified reference period. Similarly, reliability requirements are communicated for a defined reference period. In the Eurocodes reliability requirements are given for two different reference periods, i.e $T_{ref} = 1year$ and $T_{ref} = 50years$. Whereas the latter corresponds to the anticipated service life for usual structures. The introduction of two alternative reference periods is problematic, since the given reliability levels do only correspond to each other for special cases.
For $T_{ref} = 1year$ the Eurocodes specify the reliability target for RC2 with $\beta_{t,1} = 4.7$, for $T_{ref} = 50years$ it is given with $\beta_{t,50} = 3.8$.

B.2 Special case: Independent failure events and no material degradation

If it is assumed that failures (or survivals) in single years are independent from each other and no material degradation is relevant, the following simple correspondence can be established:

$$P_{f,n} = 1 - (1 - P_{f,1})^n \Leftrightarrow P_{f,1} = 1 - (1 - P_{f,50})^{1/n} \tag{16}$$

$$\beta_n = \Phi^{-1}\left(\Phi(\beta_1)^n\right) \Leftrightarrow \beta_1 = \Phi^{-1}\left(\Phi(\beta_1)^{1/n}\right) \tag{17}$$

It can be shown that for the special case of independent failure events and no material degradation the Eurocode reliability requirements for the two alternative reference periods approximately correspond to each other, i.e. $\beta_{t,50} = 3.80 \Leftrightarrow \beta_1 = 4.68 \approx 4.7 = \beta_{t,1}$.
When the design life differs from 50 years, EN1990 does not specify which target reliability to use. Two possibilities are possible: 1) keep the yearly target as a reference 2) keep the life time prob of failure. The effect of these two approaches are indicated in Figure

B.3 Dependent failure events and no material degradation

Most structural reliability problems consist of both, time dependent and time-independent random variables. The latter introduce dependency between the yearly failure (and survival) events.
For a limit state function with m time invariant variables and k time variant variables $g(x_1, \ldots, x_m, x_{m+1}, \ldots, x_{m+k})$ the correlation of failures between two different years is

$$\rho = \sum_{i=1}^{m} \alpha_i^2 \tag{18}$$

, where α_i are the so called FORM sensitivity factors of all time invariant variables. It can be seen that for correlation coefficients $\rho < 0.5$ the dependency of failure events can be neglected, i.e. assuming independence does not change the results significantly. (Note that $\rho = 0.5$ corresponds to a sum of FORM sensitivity factors of all time invariant variables of 0.71!)

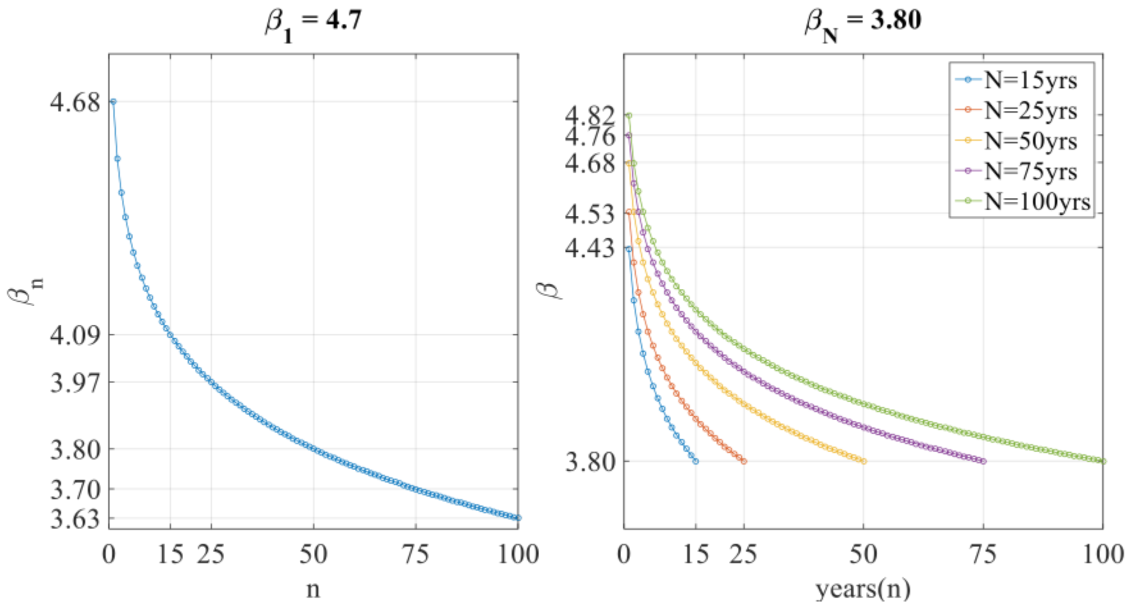

Figure 7: Left: Reliability index as function of the number of years for $\beta_1 = 4.68 \approx 4.7 = \beta_{t,1}$ (giving $\beta_{t,50} = 3.8$ as for EN1990:2002) Right: β_n giving $\beta_{t,T_{ref}} = 3.80$.

Illustrative Example

Consider a limit state function with a resistance variable R and a corresponding model uncertainty X_R, a variable representing permanent load G and a variable representing a time variable load Q with the corresponding model uncertainty X_Q:

$$g(X_R, R.G, Q, X_Q) = zX_RR - a_GG - (1 - a_G)X_QQ \leq 0 \tag{19}$$

a_G is a factor between 0 and 1 indicating the relative contribution of permanent and variable load. z is the so called design parameter.

The reliability index for different reference periods is show in the figure below for a selected case.

For the given limit state function the yearly reliability index giving $\beta_{50} = 3.8$ depends on ρ which on its turn depends on the proportion between the time-variant and time-invariant sensitivity factors.

The plots below show the variation of β_1 as a function of a and the ratio between time variant and time-invariant parts of the load for given COV for G, R and XR.

Comparison of the FORM sensitivity factors for the variable load α_G considering the yearly limit state and 50year limit state. The alpha values decreases for longer ref. periods since the COV decreases.

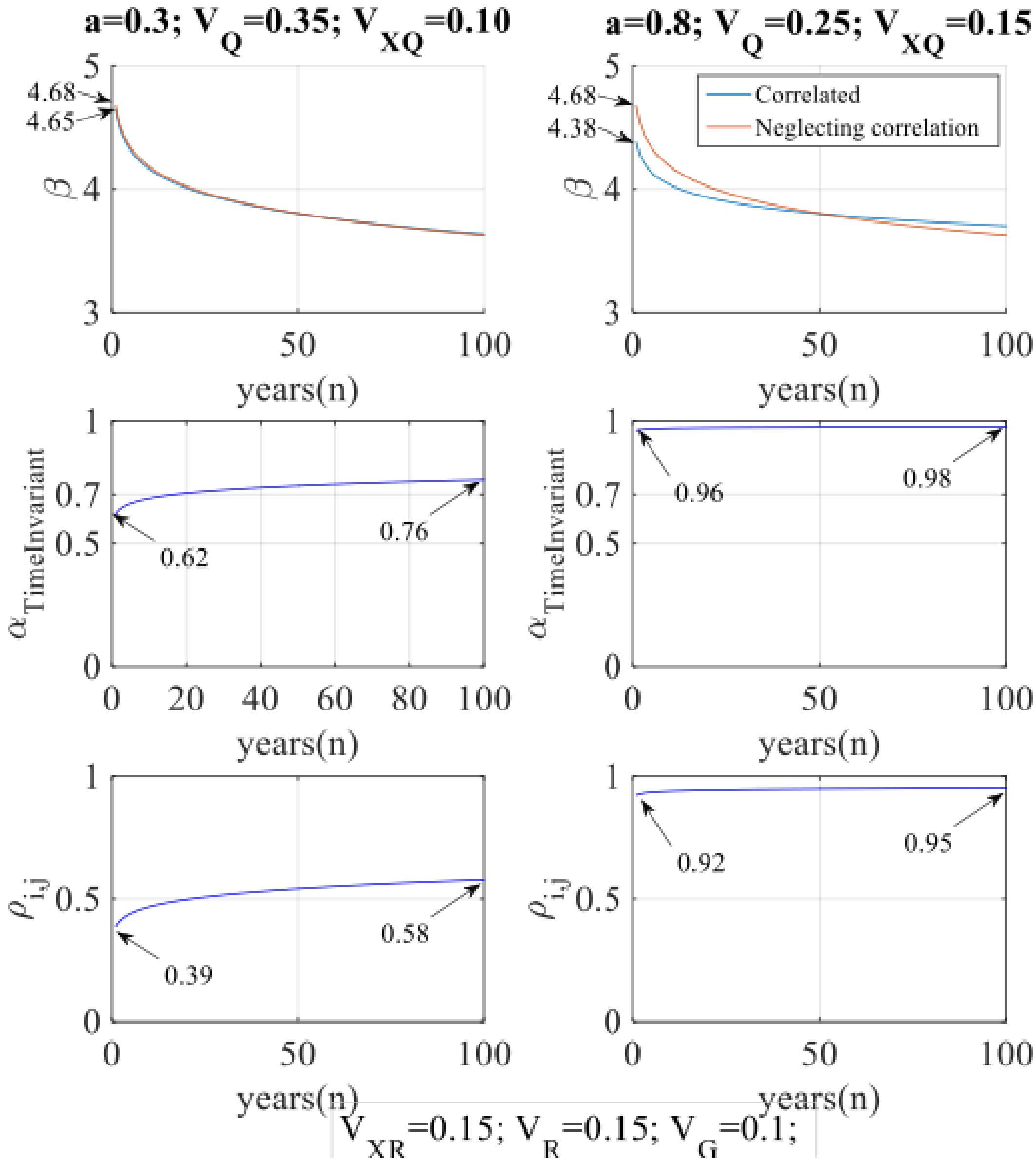

Figure 8: Structure with low *rho* as e.g. light structure with large variation of the load; Right: structure with large *rho* as e.g. heavy structure with low variation of the load (right).

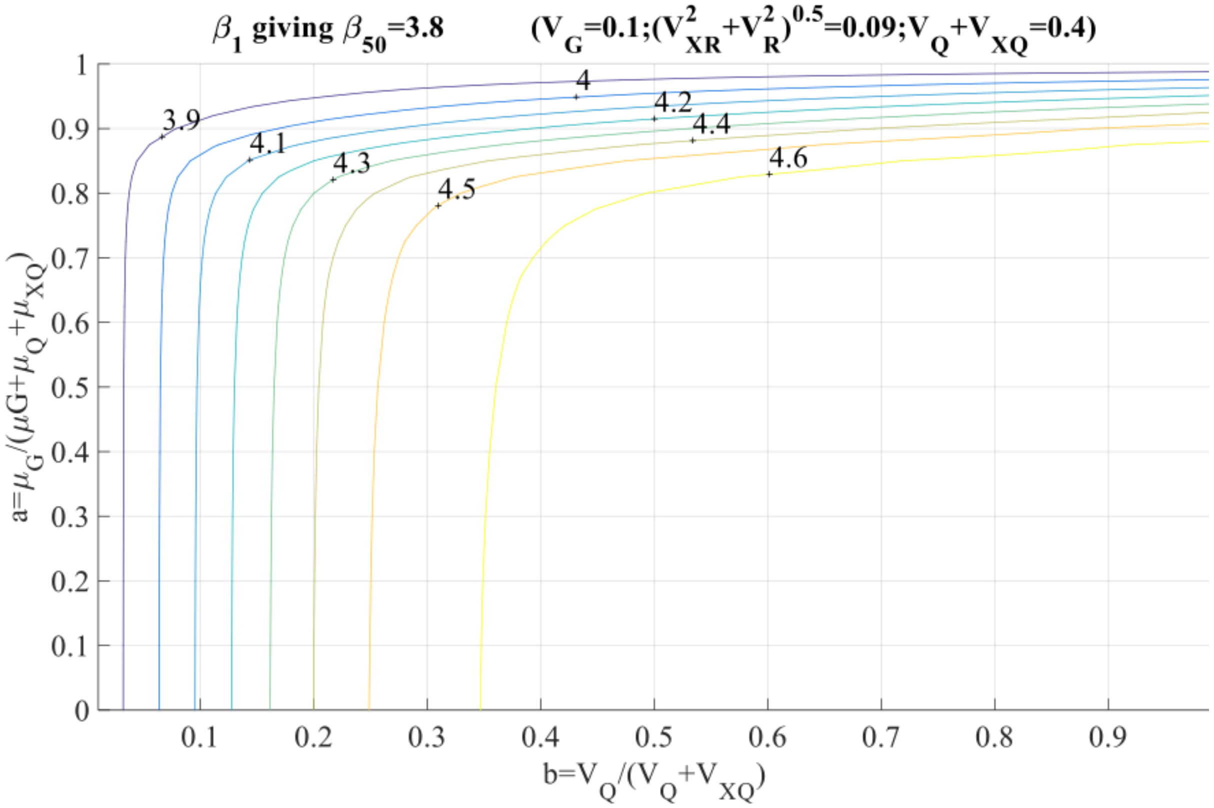

Figure 9: Yearly reliability index giving $\beta_{50} = 3.8$. (Steel + variable load with low variability)

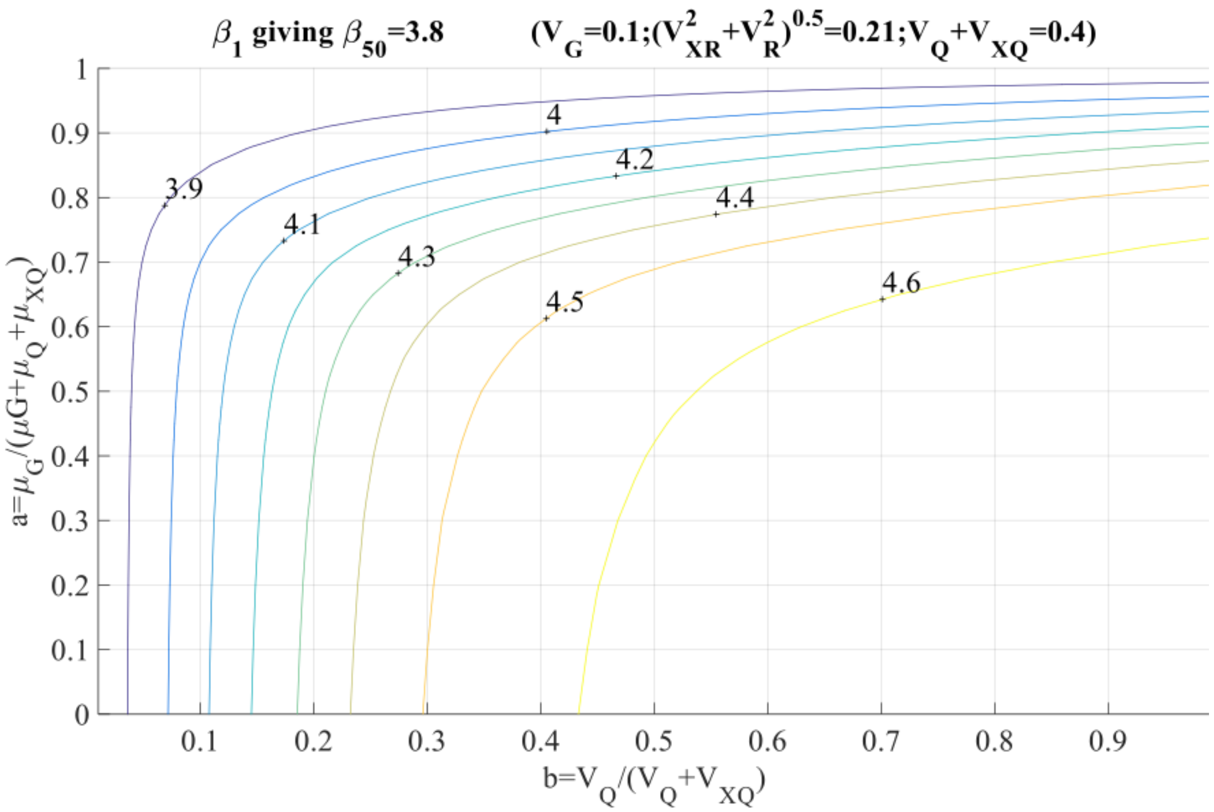

Figure 10: Yearly reliability index giving $\beta_{50} = 3.8$. (Concrete + variable load with low variability)

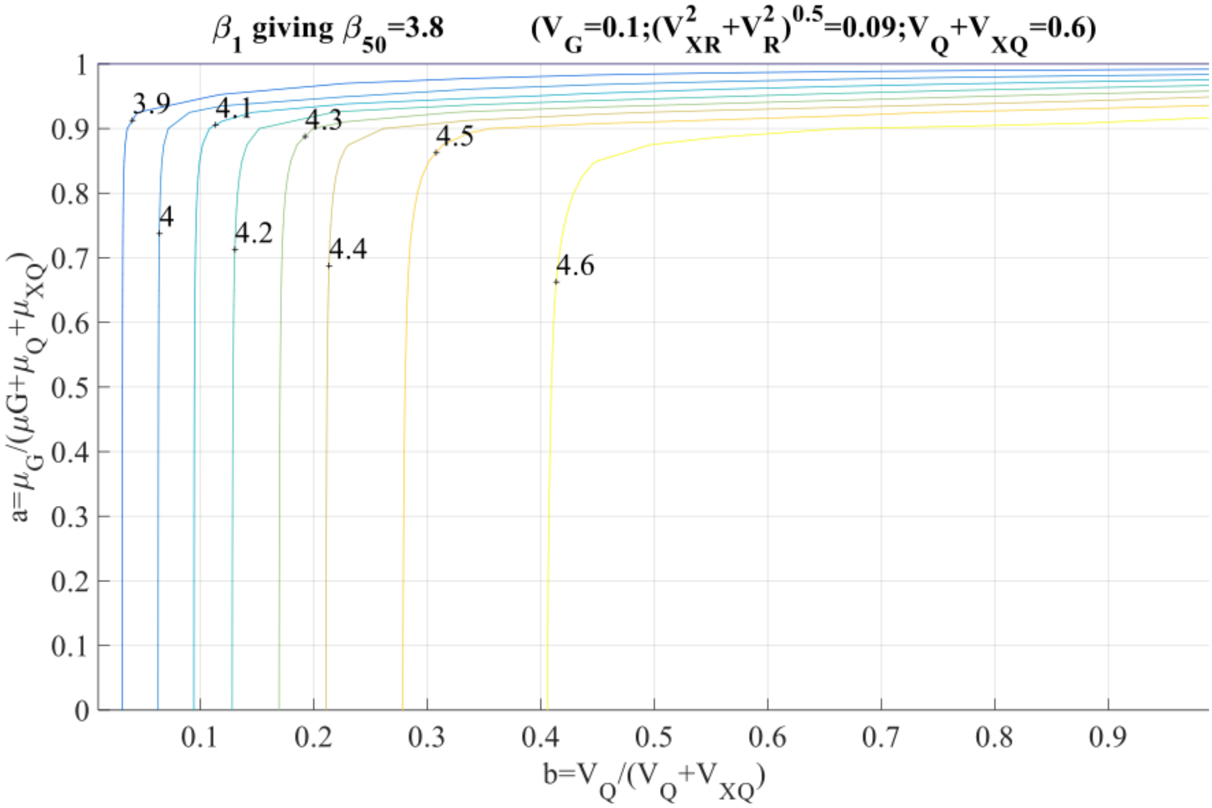

Figure 11: Yearly reliability index giving $\beta_{50} = 3.8$. (Steel + variable load with high variability)

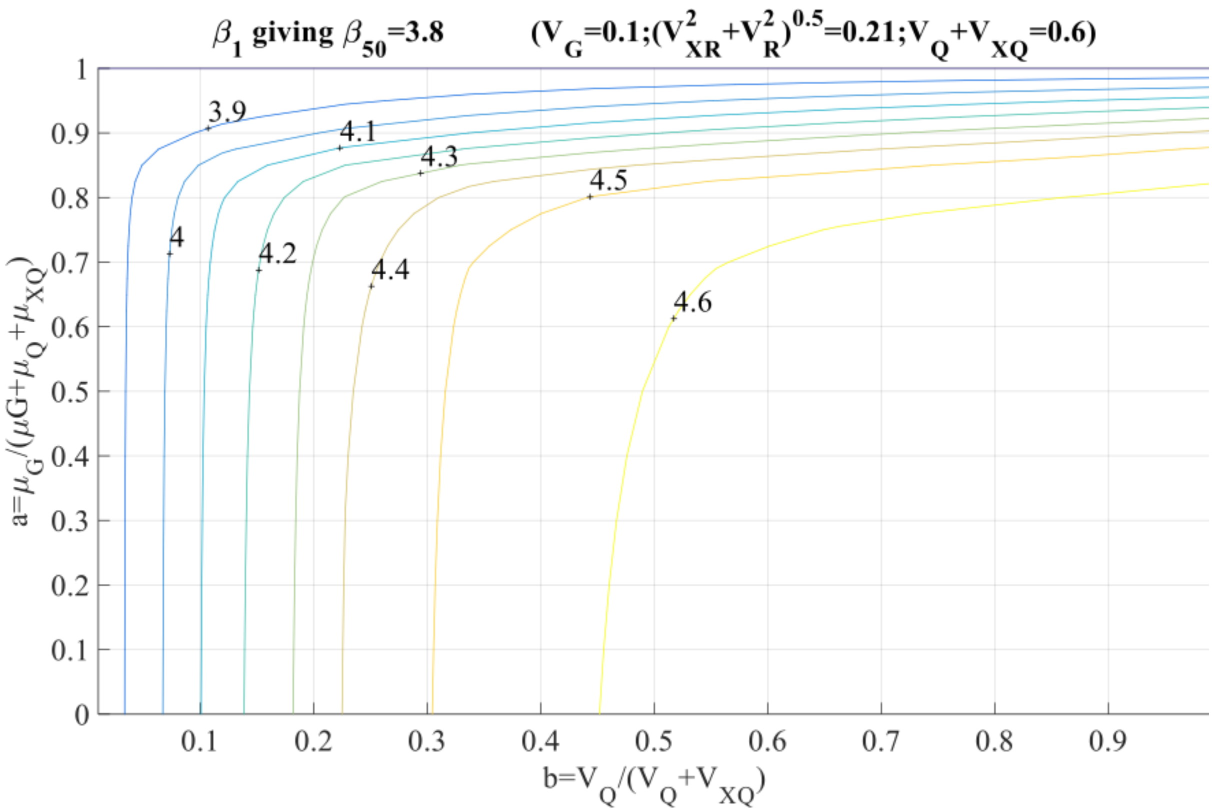

Figure 12: Yearly reliability index giving $\beta_{50} = 3.8$. (Concrete + variable load with high variability)

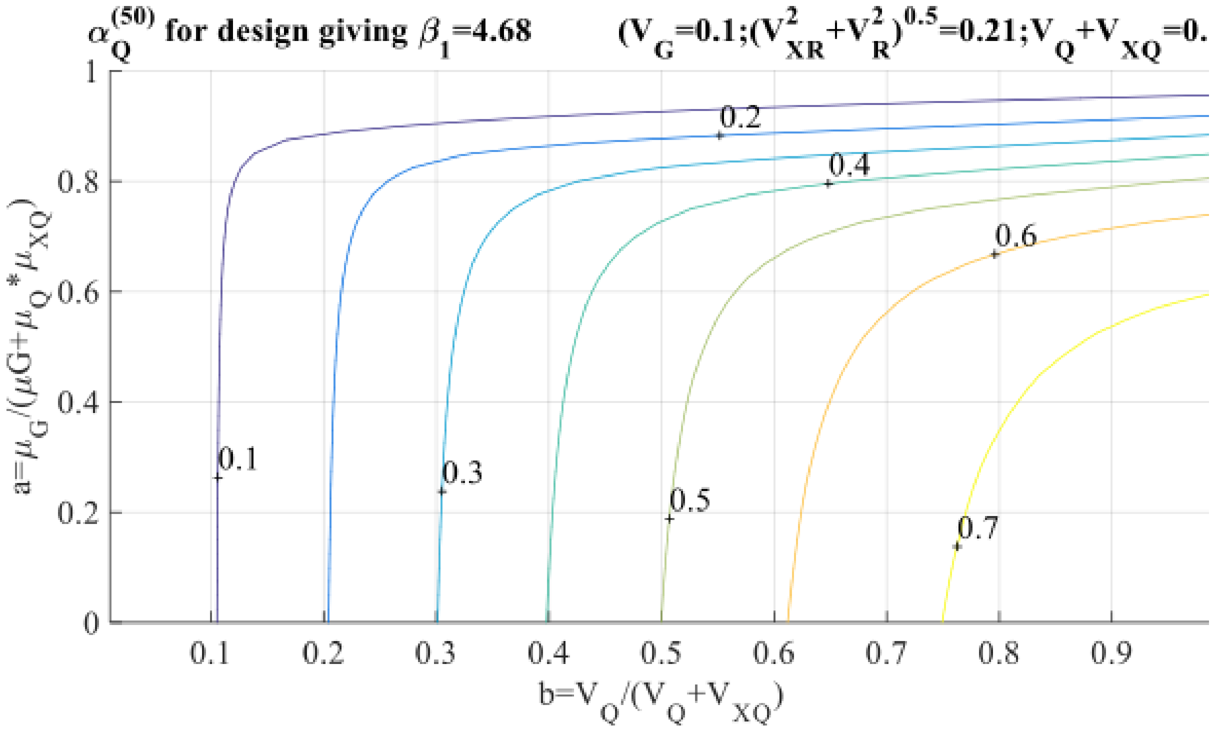

Figure 13: α_Q calculated for the limit state function with the 50 year maxima of Q and design z giving $\beta_{50} = 3.8$.

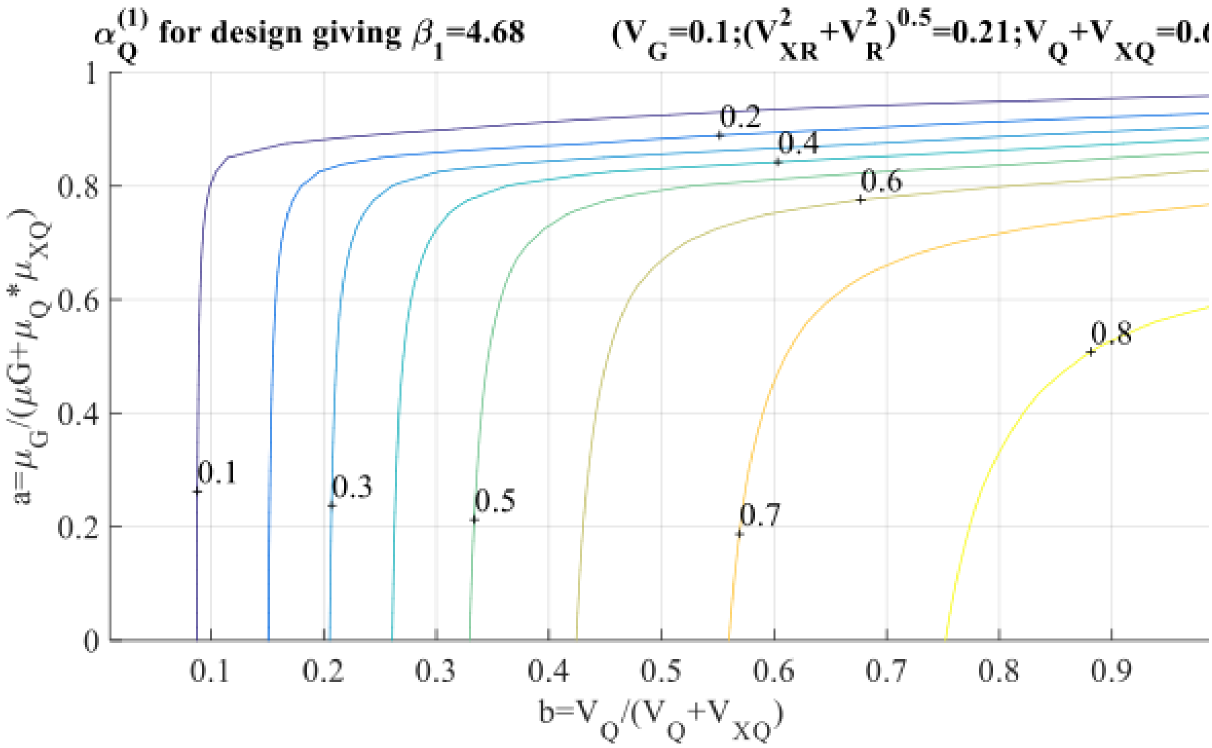

Figure 14: α_Q calculated for the limit state function with the 1 year maxima of Q and design z giving $\beta_1 = 4.7$.