

Norwegian University of Science and Technology

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Calibration of partial factor design formats

Best practice and challenge

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Context and Information

Context and Information

-
- . Getting started with Python
- . Alternatively you can access the exercise without any configuration or installation by following this link: 8 launch binder

Course Material

- Lecture Slides
- · Lecture Notes
- · Junyter Notebook

Keynote lecture

Abstart

A rational basis for the specification of reliability requirements for design and assessment of structures is introduced and discussed in this lecture. It is thereby focused on the challenges related to the practical application of reliability requirements and aspects of standardization.

- · Keynote Slides
- . Background document for further reading and reflection

This project is maintained by iochenkohler

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- Sustainable development of the build environment requires optimal balance between safety and resource efficiency.
- For structural design this balance can be identified using a high level design strategy - e.g. risk informed decision making.
- Daily life practical decisions require a simple and easy to use low level design strategy - e.g. partial factor design.

Levels of Structural Engineering Decision Making

Questions?

[Reliability based design](#page-10-0)

In the Eurocodes *appropriate* level of reliability is dependent on:

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- \cdot the possible cause and /or **mode** of attaining a limit state;
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- \cdot the possible cause and /or **mode** of attaining a limit state;
- the possible consequences of failure in terms of risk to life, injury, potential economical losses;
- \cdot public **aversion** to failure;
- the expense and procedures necessary to reduce the risk of failure.

Figure 1: Reliability requirements as stated in EN 1990:2002

Reliability based design - a simple example

$$
\begin{array}{c}\text{uniform} \\ \begin{array}{c}\nC, a \\
\downarrow S\n\end{array}\n\end{array}
$$

C,*S* Normal distributed.

Reliability based design - a simple example

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- The achieved reliability is conditional on utilised knowledge the reliability based design solution is also conditional on knowledge!
- \cdot Reliability is always dependent on specified reference time.

Questions?

[Design Value Format](#page-23-0)

Based on the simple reliability problem:

$$
\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (1)
$$

And

$$
\beta \stackrel{!}{=} \beta_{\textit{req}}
$$

Design values and characteristic values

The design value of a basic variable *Y* is defined as the multiplication or division of a corresponding partial safety factor *γ^Y* and the characteristic value *y^k* :

$$
\frac{r_k}{\gamma_R} = r_d \ge e_d = \gamma_E e_k \tag{2}
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A **characteristic value** y_k is taken as a specified *p*− fractile value from the statistical distribution $F_Y(y)$ that is chosen to represent the basic variable, as:

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y_k = F_Y^{-1}(p) \tag{3}
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Note: Typical values for *p* are:

- \cdot resistance related variables: $p = 0.05$;
- permanent actions: $p = 0.5$;
- \cdot time-variable actions (yearly reference period): $p = 0.98$.

Design value format - a simple example

$$
\begin{bmatrix} \frac{1}{2} & & & \\ & C, a & & \\ & & \downarrow S & & \end{bmatrix}
$$

C,*S* Normal distributed.

Design value format - generalisation to other distributions

Normal:
$$
y_d = \mu_Y (1 + \alpha_Y \beta_t V_Y)
$$

\n $y_k = \mu_Y (1 + \Phi^{-1}(p)V_Y)$
\nLog-Normal: $y_d = \mu_Y \exp \left(-\frac{1}{2} \ln (1 + V_Y^2) + \alpha_Y \beta_t \sqrt{\ln (1 + V_Y^2)}\right)$
\n $y_k = \mu_Y \exp \left(-\frac{1}{2} \ln (1 + V_Y^2) + \Phi^{-1}(p) \sqrt{\ln (1 + V_Y^2)}\right)$
\nGumbel: $y_d = \mu_Y \left(1 - V_Y \frac{\sqrt{6}}{\pi} (0.5772 + \ln (-\ln (\Phi(\alpha_Y \beta_t))))\right)$
\n $y_k = \mu_Y \left(1 - V_Y \frac{\sqrt{6}}{\pi} (0.5772 + \ln (-\ln (p))))\right)$

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- ... but only for specific design cases.
- \cdot The α values are case specific and their determination may be cumbersome.
- \cdot Both, α and the extreme value distribution representing the variable load have to relate to the same time reference period than the reliability target.

Questions?

[Generalising](#page-35-0) *α*

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- For ease of practical application, it would be good to prescribe a set of generalised *α* values.
- \cdot The set of generalised α values shall lead to **safe** design solutions **for most** of the cases.
- Alternative representation of the reliability problem for an informed choice.

Hashofer-Lind representation of reliability problem

Hashofer-Lind representation of reliability problem

Hashofer-Lind representation of reliability problem

C,*S* Normal distributed.

Generalisation

The following Eurocode standardized values can be used for a 50 years reference period:

- If *Y* represents a strength related variable: $\alpha_Y = -0.8$
- \cdot If *Y* represents a load related variable: $\alpha_Y = 0.7$
- \cdot If *Y* is dominating the reliability problem: α _{*Y*} = (−)1
- If *Y* represents a secondary strength or load related variable: *α^Y* = *−*0*.*8 *·* 0*.*4 or $\alpha_Y = 0.7 \cdot 0.4$ correspondingly.

[Reality check - extended examples](#page-44-0)

Initial Example continued

ШШШШ $\angle C, a$ \downarrow S

Initial Example continued

ШШШШ $\angle C, a$ \downarrow S

Ext. Example - Application of the generalized *α*- values

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A simple calibration case study

$$
H(R, G, Q, X_Q) = zR_i - (1 - a)G - aX_QQ \quad \text{with}
$$
\n
$$
z = \gamma_{R_i} \frac{(1 - a) \cdot \gamma_G \cdot g_k + a \cdot \gamma_Q \cdot q_k^*}{r_{R,i}}
$$

(4)

A simple calibration case study

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$$

 $Q^* = X_Q Q_{1a}$ and q^*_k such that $F_{Q^*}(q^*_k) = 0.98$

(4)

A simple calibration case study - real alpha values

A simple calibration case study - generalized alpha values

A simple calibration case study - generalized alpha values applied on material

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	- \cdot unsafe, i.e. achieved reliability is below the reliability requirement,
	- safe by large margin, that corresponds to unnecessary use of material.
- \cdot Especially the application of the generalised α -value on single variables in isolation is not effective and, as demonstrated in this note, the obtained safety levels are partly not acceptable.
- It is recommended to reconsider the recommendation of the design value approach with its generalised *α*-values in the revision of the Eurocodes.

Questions?

[Alternative approach to calibration](#page-61-0)

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- We search for the *best compromise*.

$$
\min_{\gamma_R^*, \gamma_G^*, \gamma_Q^*} \left\{ \sum_{i=1}^n \left(\beta_t - \beta_i(\gamma_R, \gamma_G, \gamma_Q, \mathcal{D}_i) \right)^2 \right\} \tag{5}
$$

Calibration as an optimisation problem

- Partial factors to be applied for a domain of design situations.
- We search for the *best compromise*.
- The best compromise to be identified by simple least square difference to the target, as

$$
\min_{\gamma_R^*, \gamma_G^*, \gamma_Q^*} \left\{ \sum_{i=1}^n \left(\beta_t - \beta_i (\gamma_R, \gamma_G, \gamma_Q, \mathcal{D}_i) \right)^2 \right\} \tag{5}
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Questions?