



Innovation and Networking for Fatigue and  
Reliability Analysis of Structures – Training for  
Assessment of Risk



# Design and assessment criteria for safety and cost efficiency

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Coordinated by



Université  
Gustave Eiffel

# Overview

- General performance criteria for the build environment
- Self-contained approach to assess performance
- Simplification = Generalisation
- How safe is safe enough? => A calibration problem!

# The build environment

- As the main contributor to our societal development,
- And, as a major consumer of natural resources,
- Needs proper **strategies for decision support** for further development and maintenance !!
- Objective: sustainable development.

# The build environment



# Strategy

- Decisions are made
- It is not how we can identify the right decision, but how we identify the “best” decision
- Reasonable to assess the effect of different decision alternatives on “our” utility

# Formal Decision Theory

Actions

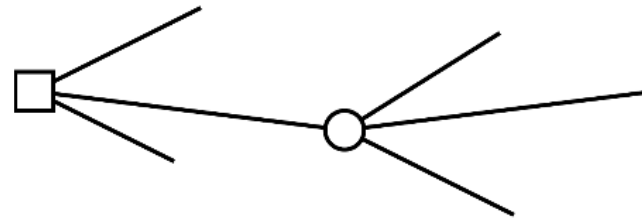
$a \in A$

State

$\theta \in \Theta$

Utility

$u(\cdot)$



What can I know?

What should I do?

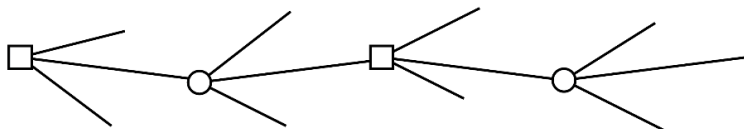
What may I hope?

- Reasonable strategy
- Challenging to apply
- Simplifications necessary

# System definition

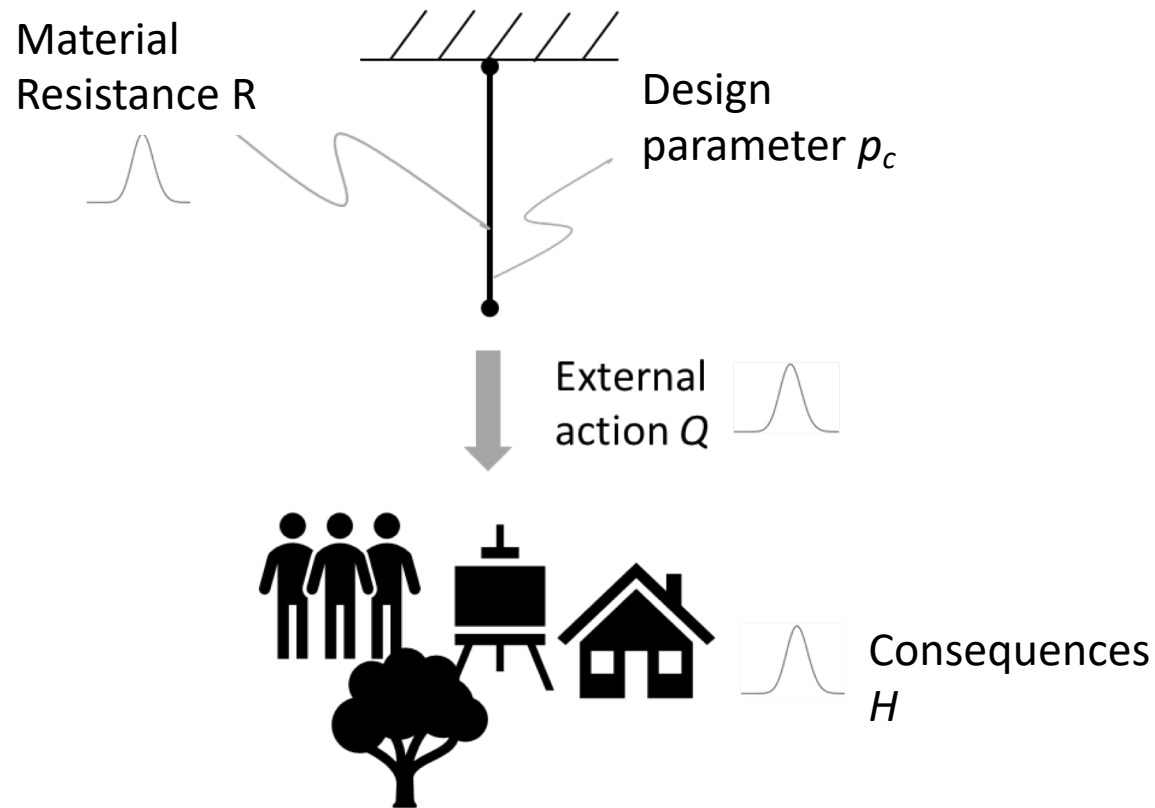
- Reduction and simplification
- Decision alternatives  $\leftrightarrow$  utility

| <u>Experiment</u> | <u>Sample</u> | <u>Actions</u> | <u>State</u>        | <u>Utility</u> |
|-------------------|---------------|----------------|---------------------|----------------|
| $e \in E$         | $z \in Z$     | $a \in A$      | $\theta \in \Theta$ | $u(.)$         |



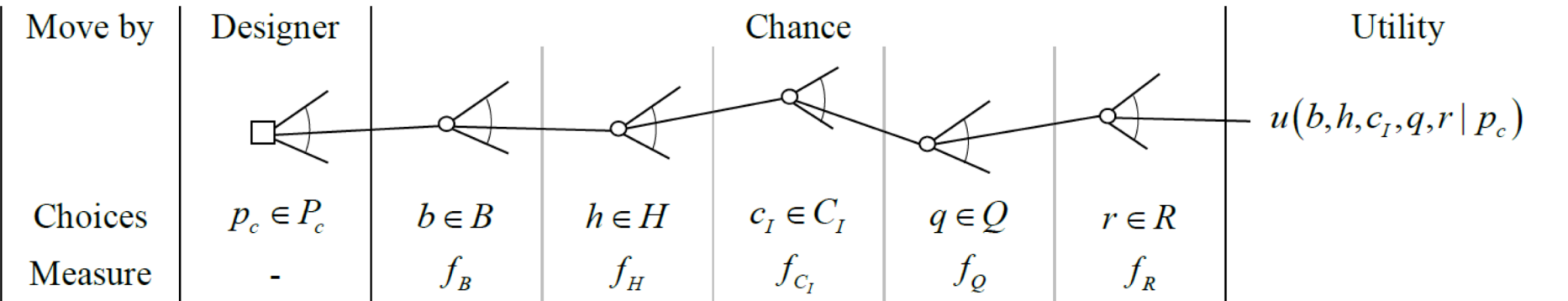
# Structural design decision problem

- Objective: minimum use of resources over time





# Structural design decision

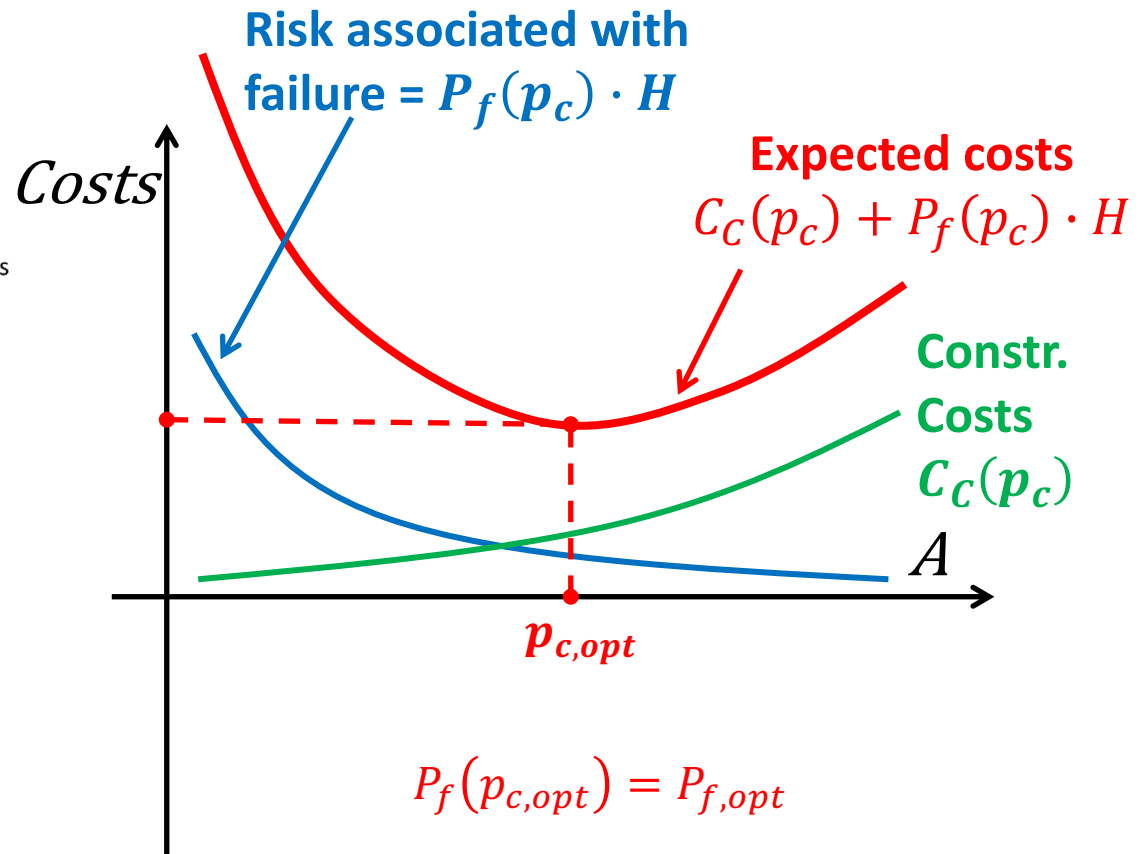
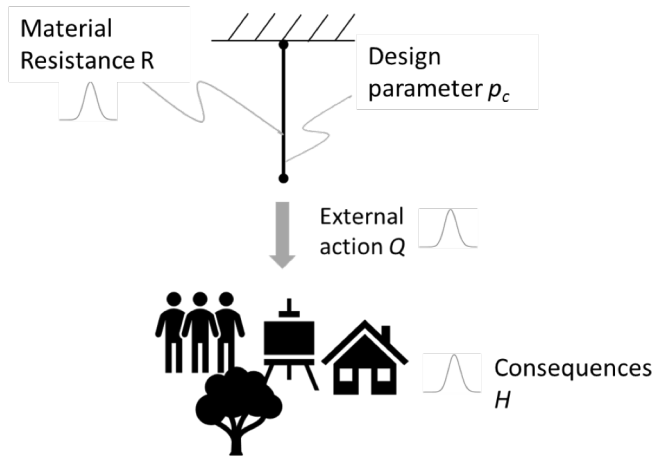


$$\mathbf{p}_{c,opt} = \underset{\mathbf{p}_c}{\operatorname{argmax}} \{E_{\Theta}[u(\Theta, \mathbf{p}_c)]\} = \underset{\mathbf{p}_c}{\operatorname{argmin}} \{E_{\Theta}[C_{tot}(\Theta, \mathbf{p}_c)]\}$$

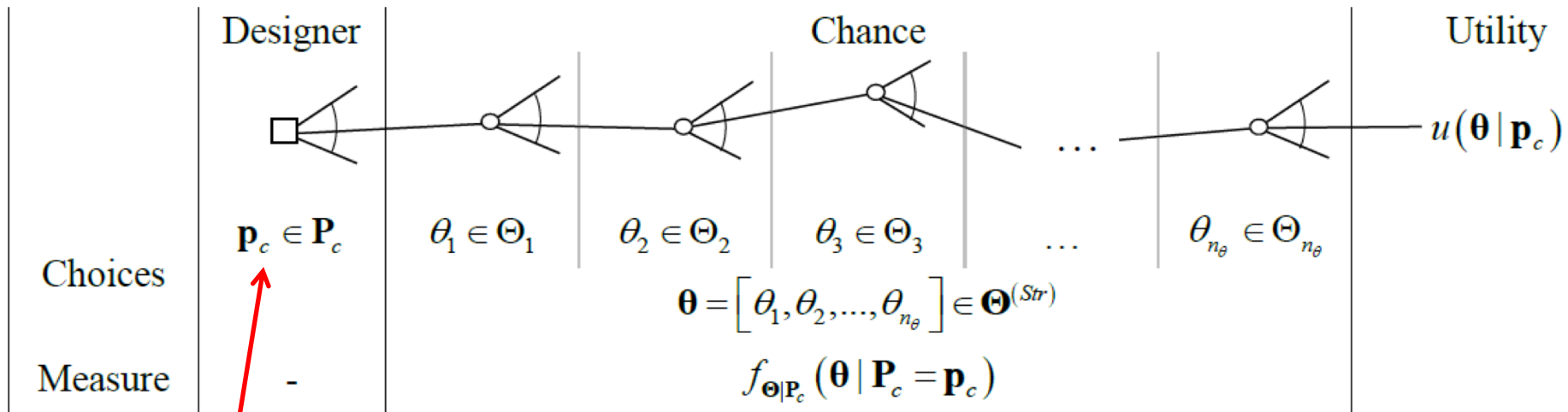
$$E_{\Theta}[C_{tot}(\Theta, \mathbf{p}_c)] = (E[C_0] + E[C_1]p_c) - E[H]P_f(\mathbf{p}_c)$$

$$P_f(\mathbf{p}_c) = \int_{p_c r < q} f_{R,Q}(r, q) dr dq$$

# Risk informed decision



# Generalization of the risk informed design problem



Decision variable

"State of the world" Variables  
Not known with certainty by the decision  
maker

Level of detail  
sensibly chosen!

$$\mathbf{p}_{c,opt} = \operatorname{argmax}_{\mathbf{p}_c} \{E_{\boldsymbol{\theta}}[u(\boldsymbol{\theta} | \mathbf{p}_c)]\}$$

# Simplified design methods

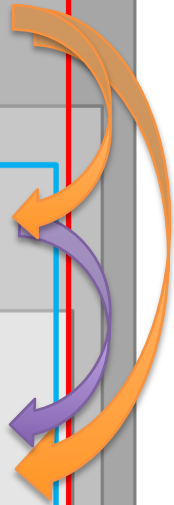
| Approaches:  | Simplifications:  | Objective:  |
|--|---|---|
| <b>Risk-informed</b><br>Decisions taken considering full risk (Level 4 design )  | None  | <i>Minimise use of societal resources over time</i>                       |
| <b>Reliability-based</b><br>Decisions taken with reliability requirement to fulfil (Level 3 and 2 design)                            | Avoid explicit evaluation of failure consequences/ safety costs etc.                                | Target reliability index or Pf  |
| <b>Semi-probabilistic</b><br>Safety format prescribing the design equations and/or analysis for assessing decisions (Level 1 design) | Avoid explicit evaluation of failure consequences/ safety costs etc. AND avoid reliability analyses | Partial safety factors, modification factors, load reduction factors etc. |

Application domains

Reliability elements in standards:

Reliability-based calibration

Risk-based calibration

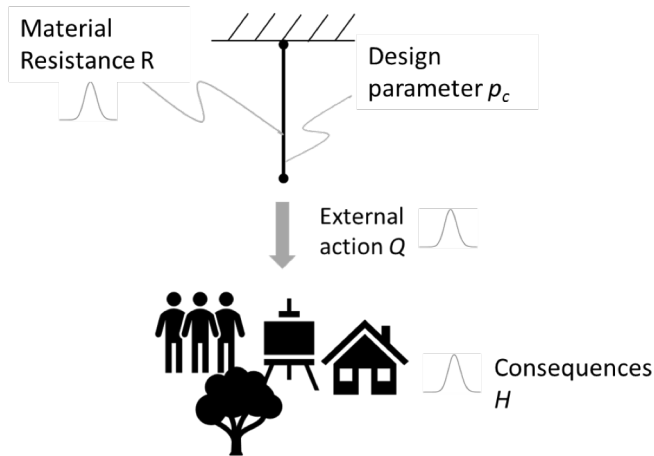


# Simplified design and assessment of decision approaches [ISO 2394]

- **Level 4: Risk-informed**
- **Levels 3 (and 2): Reliability-based**



# Reliability based design (Level 3 and 2)



Design:

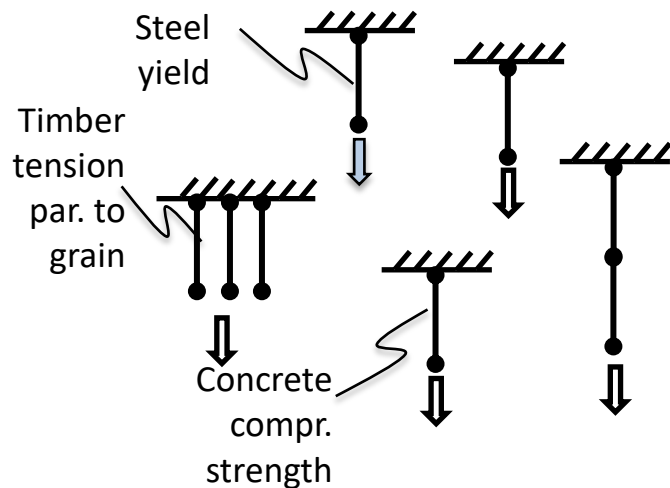
$$p_c: P_f(p_c) = P_{f,target}$$

$$\text{Level 3} \equiv \text{Level 4} \Leftrightarrow P_{f,target} \equiv P_{f,opt}$$

# Code calibration, why?

- Simplification:
  1. No explicit evaluation of costs, consequences, etc

→ simplify calculations

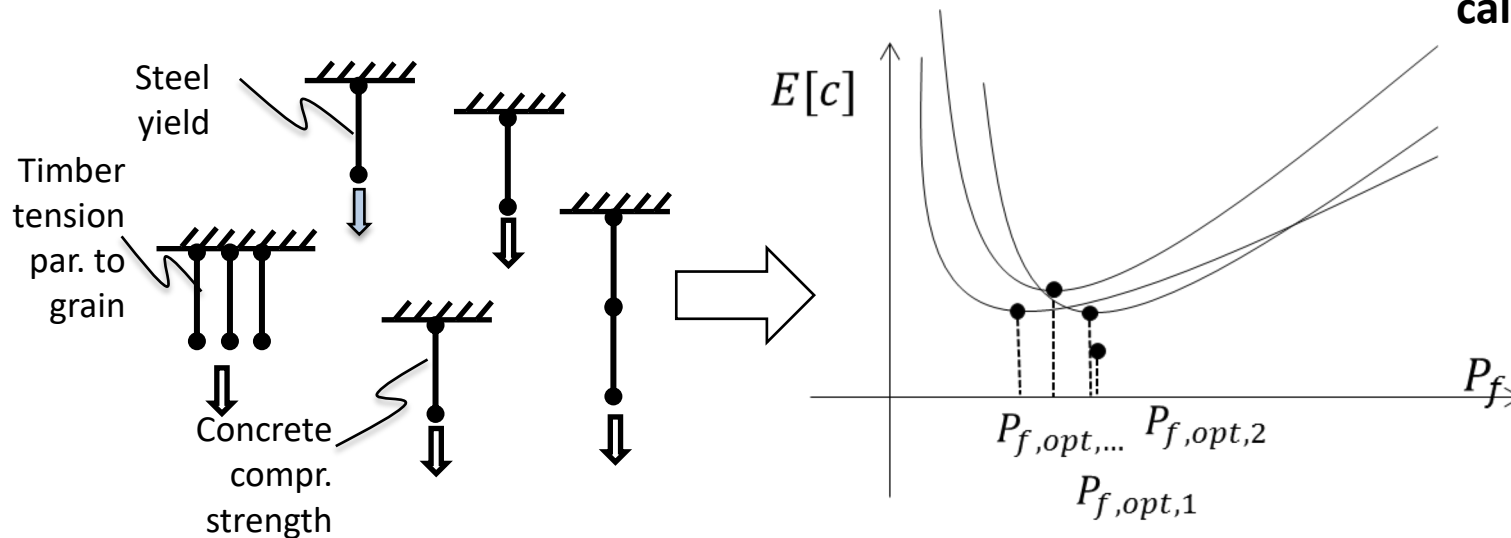


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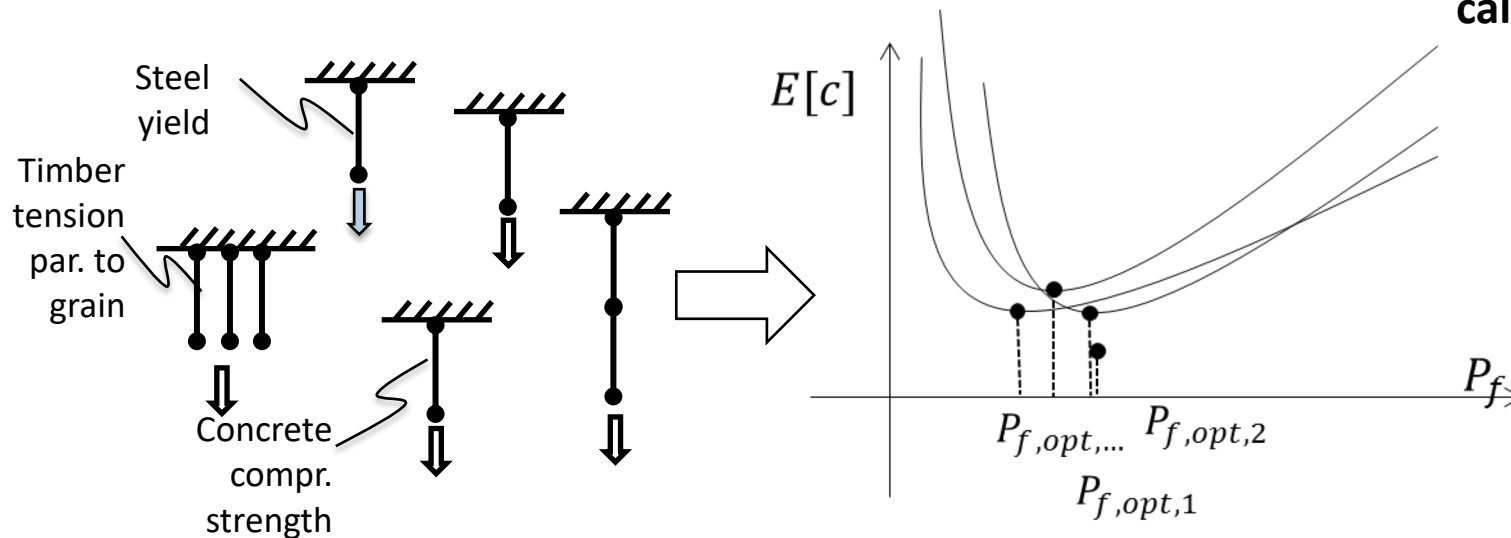


# Code calibration, why?

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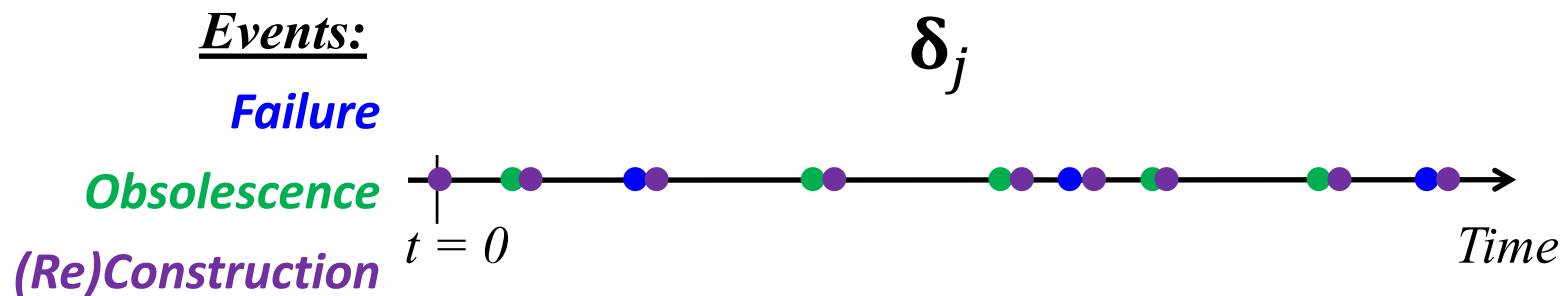
2. One  $P_{f,target}$  for a class of structures

→ simplify standards and calculations

**CALIBRATION: what  $P_{f,target}$  is optimal for the class?**

# Code calibration as a decision problem under risk

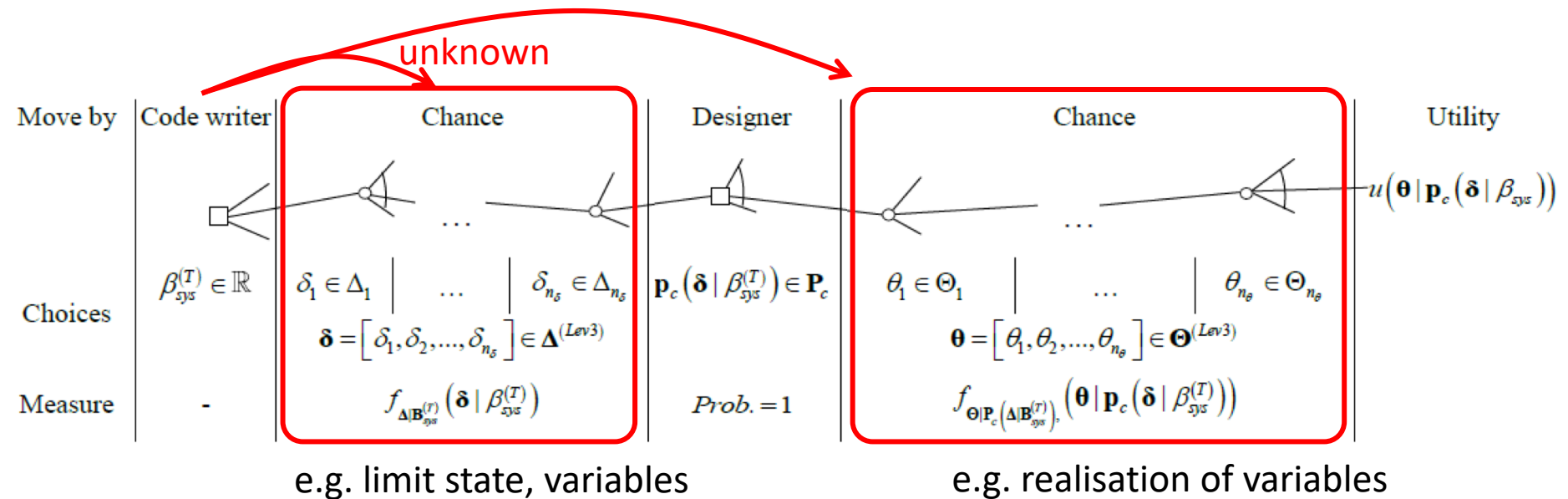
- Decision variable:  $\beta_{target}$  for Level 3 and 2 design
  - each structure in the class defined by  $\delta$
  - present and future structures



- Decision maker: society (codes guard the interest of society)
- Level of detail in system representation consistent with the generalisation over classes

# Optimisation of $\beta_t$ for Level 3 codes

- Game between *Code writer* and *Chance*
  1. *Code writer* selects a  $\beta_t$
  2. *Chance* chooses a possible structure to be designed  $\delta \in \Delta^{(Lev3)}$
  3. Designer finds dimensions  $\mathbf{p}_c$  giving  $\beta \equiv \beta_t$
  4. *Chance* chooses a state of the nature  $\theta \in \Theta^{(Lev3)}$

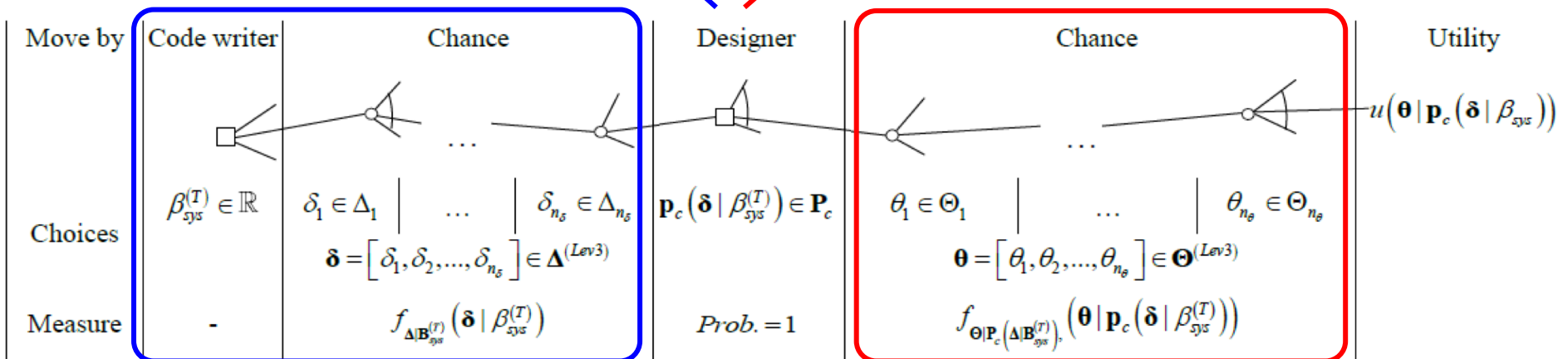


# Optimisation of $\beta_{sys,t}$ for Level 3 codes

- Game between *Code writer* and *Chance*
  1. *Code writer* selects a  $\beta_t$
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  3. Designer finds dimensions  $\mathbf{p}_c$  giving  $\beta \equiv \beta_t$
  4. *Chance* chooses a state of the nature  $\theta \in \Theta^{(Lev3)}$

Known and accounted in design

unknown



e.g. limit state, variables

e.g. realisation of variables

# Current target reliability values in JCSS PMC and ISO 2394

- Based on monetary optimization

|                         |        | Failure consequences          |                                       |                                       |
|-------------------------|--------|-------------------------------|---------------------------------------|---------------------------------------|
|                         |        | Minor                         | Moderate                              | Large                                 |
| Relative cost of safety | Large  | 3.1 ( $P_f \approx 10^{-3}$ ) | 3.3 ( $P_f \approx 5 \cdot 10^{-4}$ ) | 3.7 ( $P_f \approx 10^{-4}$ )         |
|                         | Normal | 3.7 ( $P_f \approx 10^{-4}$ ) | 4.2 ( $P_f \approx 10^{-5}$ )         | 4.4 ( $P_f \approx 5 \cdot 10^{-6}$ ) |
|                         | Small  | 4.2 ( $P_f \approx 10^{-5}$ ) | 4.4 ( $P_f \approx 5 \cdot 10^{-6}$ ) | 4.7 ( $P_f \approx 10^{-6}$ )         |

- Risk optimisation philosophy included by differentiation of consequences and cost for safety.
- Differentiation is coarse - > consistent with level of information.
- But qualification into classes is difficult.

# Background Reliability Target Table

- Objective function

$$\begin{aligned} E [C_{tot} (p)] &= C_{constr} (p) + E [C_f (p)] \frac{1}{\gamma} + E [C_{obs} (p)] \frac{1}{\gamma} \\ &= [C_0 + C_{IP}] + [C_0 + C_{IP} + H] \frac{\lambda P_f^{(1a)} (p)}{\gamma} + [C_0 + C_{IP} + D] \frac{\omega}{\gamma} \end{aligned}$$

- Yearly probability of failure based on the simple  $R - S$  problem.
- The variability of  $R$  and  $S$  chosen such that it represents the characteristics of a class of structures.

# Background Reliability Target Table

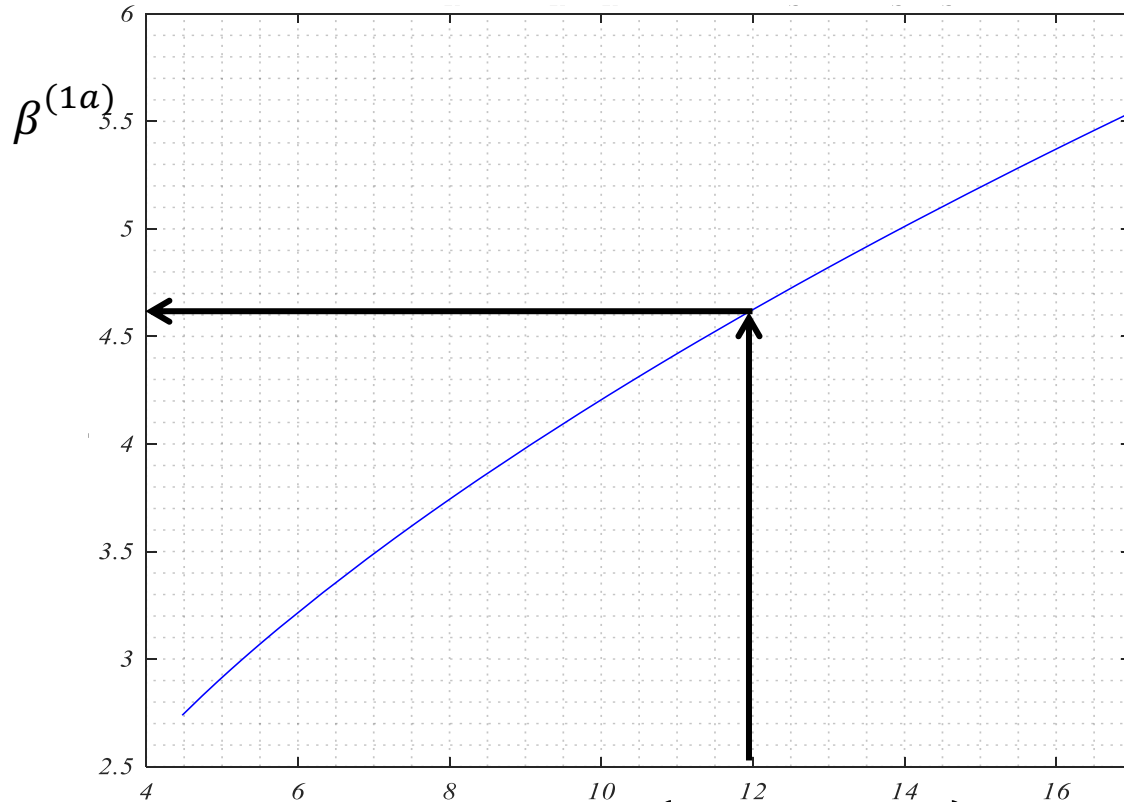
- Optimisation

$$\frac{d}{dp} \left\{ C_0 + C_I p + [C_0 + C_I p + H] \frac{\lambda P_f^{(1a)}(p)}{\gamma} + [C_0 + C_I p + D] \frac{\omega}{\gamma} \right\} \Bigg|_{p=p^*} \equiv 0$$

$$\Rightarrow \frac{C_0 + C_I p^* + H}{C_I} = \frac{1 + P_f^{(1a)}(p^*) \frac{1}{\gamma} + \frac{\omega}{\gamma}}{-\frac{dP_f^{(1a)}(p)}{dp} \Big|_{p=p^*} \frac{1}{\gamma}}$$

- Reordering and simplification: 
$$\frac{C_I \cdot (\gamma + \omega)}{C_0 + H} \approx -\frac{dP_f^{(1a)}(p^*)}{dp} \Big|_{p=p^*}$$

# Plot representing target reliabilities



Line satisfying the condition at optimum

$$\frac{C_I \cdot (\gamma + \omega)}{C_0 + H} \approx - \left. \frac{dP_f^{(1a)}(p^*)}{dp} \right|_{p=p^*}$$

for:

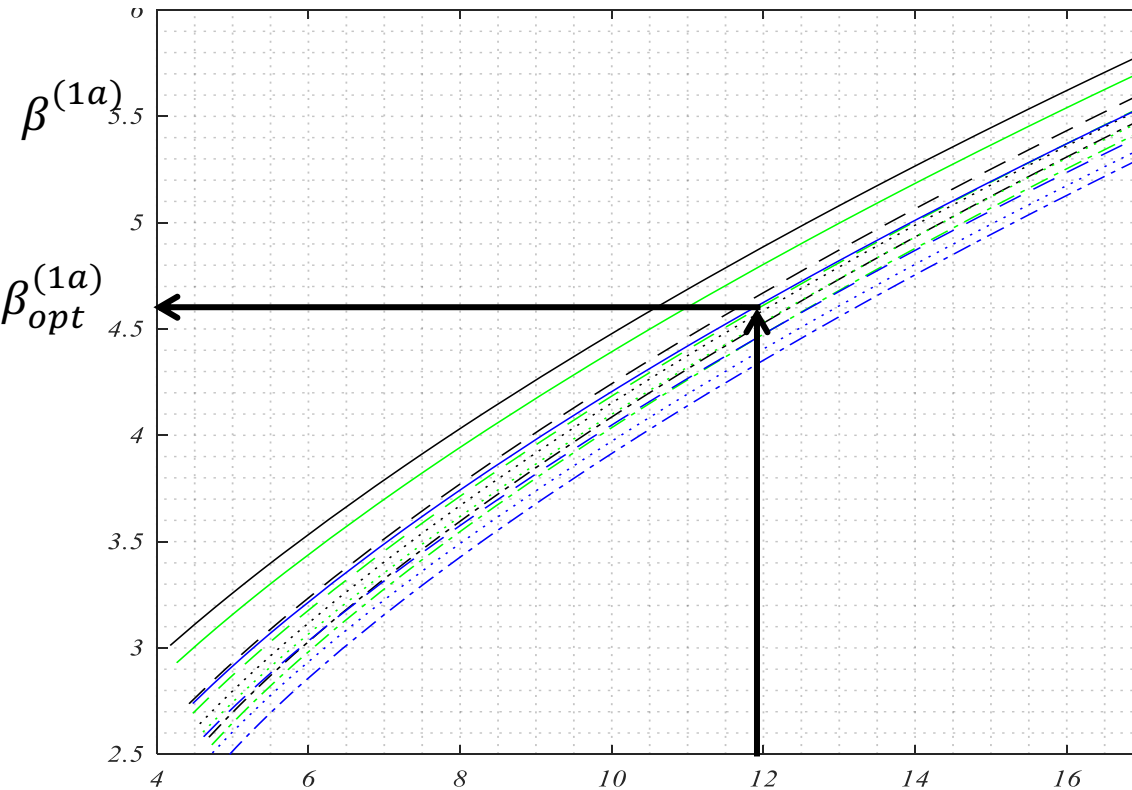
$$COV_R = 0.15 \text{ and } COV_S = 0.30$$

$$\ln \left\{ \frac{C_0 + H}{C_I(\gamma^{(1a)} + \omega)} \right\}$$

Safety costs;  
Failure costs;  
Interest rate  $\gamma$ ;  
Obsolescence rate  $\omega$ .



# Plot representing target reliabilities



## Different types of uncertainties

- $V_R=0.05, V_S=0.1$
- - -  $V_R=0.05, V_S=0.3$
- .....  $V_R=0.05, V_S=0.45$
- - - -  $V_R=0.05, V_S=0.6$
- $V_R=0.15, V_S=0.1$
- - -  $V_R=0.15, V_S=0.3$
- .....  $V_R=0.15, V_S=0.45$
- - - -  $V_R=0.15, V_S=0.6$
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- - - -  $V_R=0.3, V_S=0.6$

$$\ln \left\{ \frac{C_0 + H}{C_I(\gamma^{(1a)} + \omega)} \right\}$$

Safety costs;  
Failure costs;  
Interest rate  $\gamma$ ;  
Obsolescence rate  $\omega$ .

# Life Safety

- The reliability requirement, so far, was based on optimisation.
- Our societal preferences for life safety can not be related to potential benefit of a economic endeavour!
- On the other hand, additional reliability is obtained by investing more monetary means.
- Societal willingness to pay (SWTP): How much **can** a society invest to reduce the fatality rate in structures?

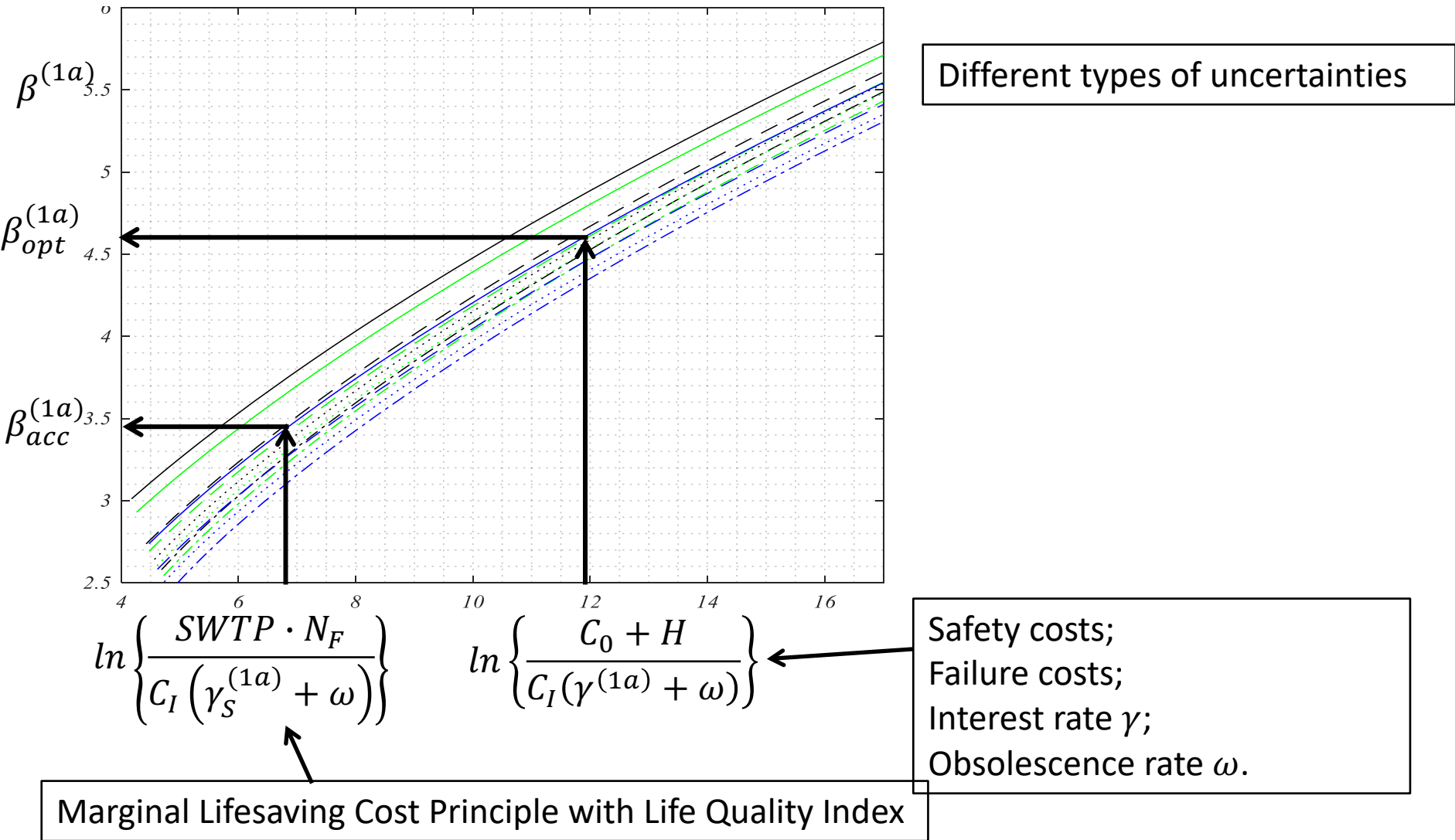
# Life Safety – modified objective

$$\frac{d}{dp} \left\{ C_0 + C_I p + N_F SWTP \frac{\lambda P_f^{(1a)}(p)}{\gamma} + [C_0 + C_I p + D] \frac{\omega}{\gamma} \right\} \Big|_{p=p^*} \equiv 0$$

- Correspondingly it has to be invested at least:

$$-\frac{dP_f^{(1a)}(p)}{dp} \leq \frac{C_I (\gamma_S + \omega)}{SWTP \cdot N_F} = K_1$$

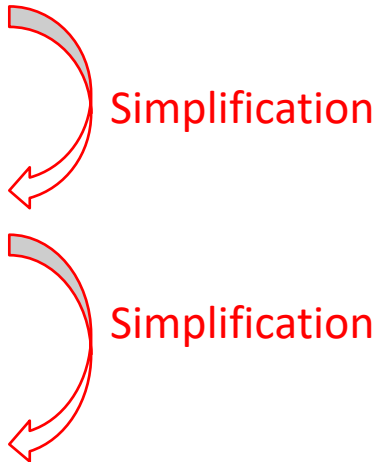
# Plot representing target reliabilities



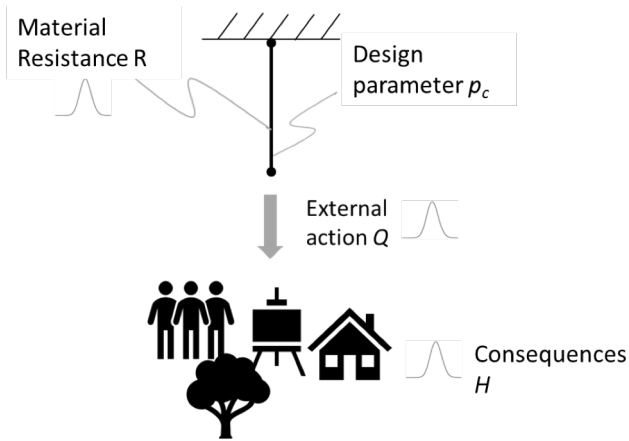
# Summary

- Determination of target reliabilities for reliability based design is a calibration problem
- Generalisation and classification requires “low” level of detail of system representation
- Risk criteria can be in-cooperated
- Risk based design is open to any/(the appropriate) level of detail.

# Simplified design and assessment of decision approaches [ISO 2394]

- **Level 4: Risk-informed**
  - **Levels 3 (and 2): Reliability-based**
  - **Level 1: Semi-probabilistic**
- 
- The diagram illustrates the relationship between the levels of decision approaches. It features two red curved arrows pointing from the right towards the text. The top arrow points from the word 'Simplification' to the text 'Levels 3 (and 2): Reliability-based'. The bottom arrow points from the word 'Simplification' to the text 'Level 1: Semi-probabilistic'. This indicates that simplification is applied to move from higher levels (3 and 2) to lower levels (1).

# Semi-probabilistic approach (Level 1)



Partial Safety Factors  
(reliability elements)

**Design:**

$$p_c: p_c \geq \frac{\gamma_M}{f_k} \cdot \gamma_Q \cdot q_k$$

$$\text{Level 1} \equiv \text{Level 4} \Leftrightarrow \gamma_M, \gamma_Q: P_f \left( p_c = \frac{\gamma_M}{f_k} \cdot \gamma_Q \cdot q_k \right) \equiv P_{f,opt}$$

# Code calibration, why?

- Simplification:

1. No explicit evaluation costs, consequences, etc.
2. No reliability analyses

→ **simplify  
calculations**

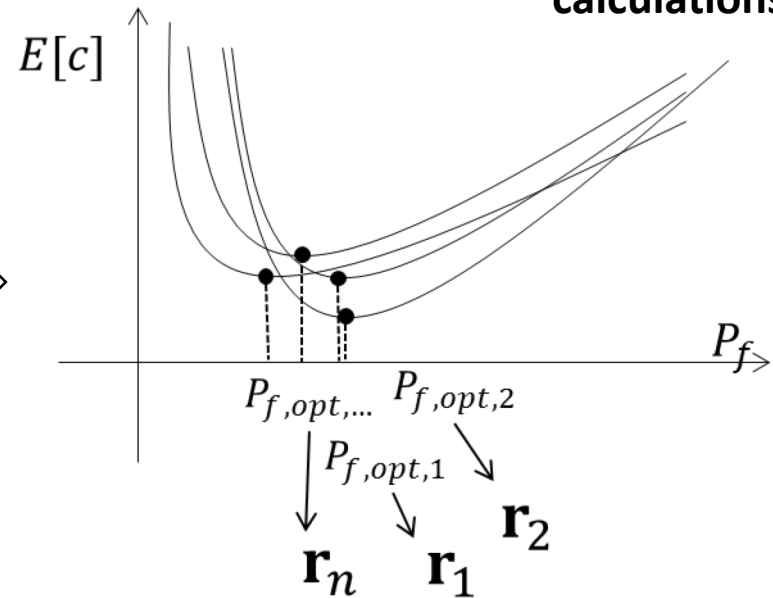
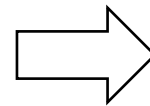
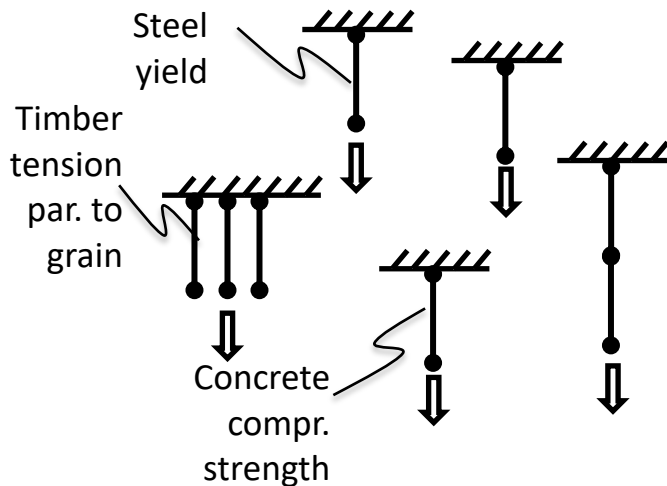


# Code calibration, why?

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→ simplify calculations

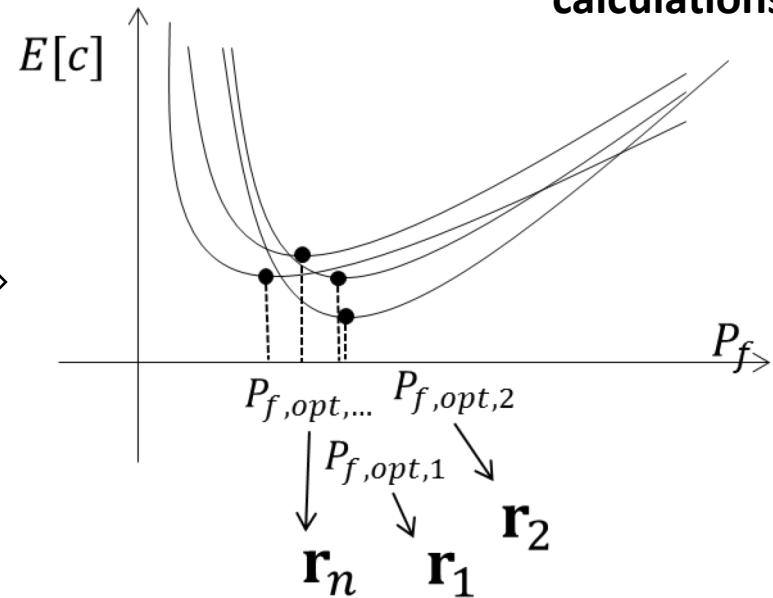
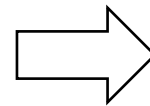
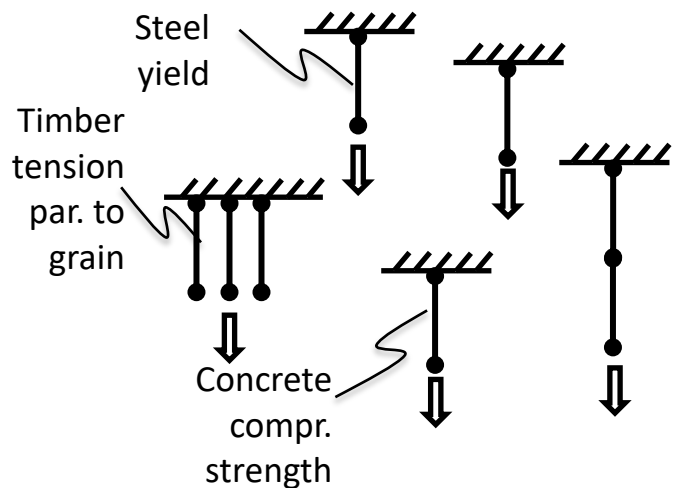


# Code calibration, why?

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→ simplify calculations



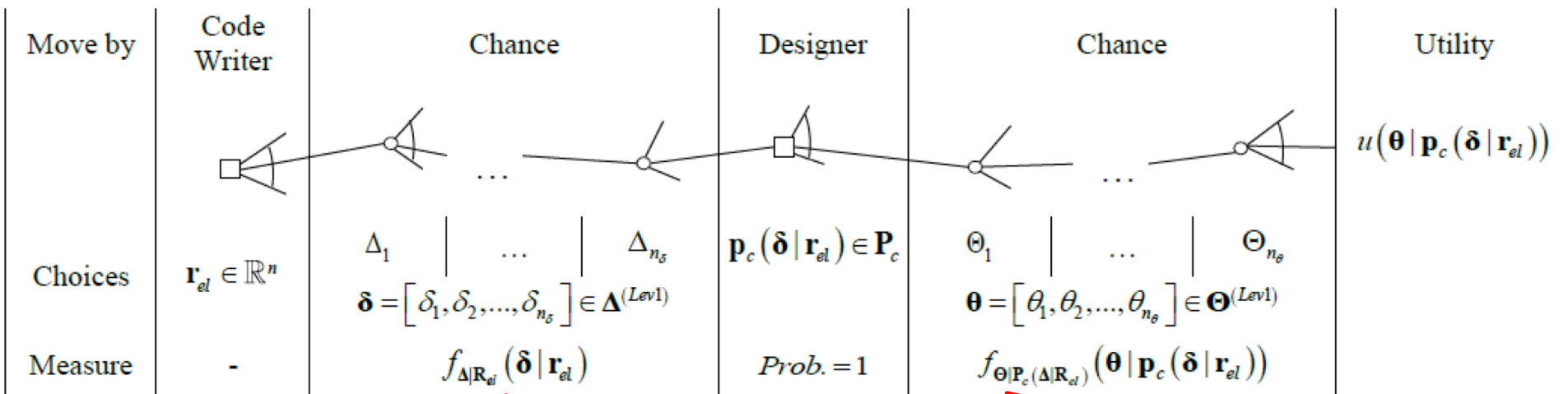
3. One  $\mathbf{r} = [\gamma, \psi_0, k_{mod}]$  for a class of structures

→ simplify standards and calculations

**CALIBRATION: what  $\mathbf{r}$  is optimal for the class?**

# Decision problem

- Decision variable:  $\mathbf{r}_{el}$  for Level 1 design for a class of structures
  - Partial safety factors
  - Modification factors
  - Load combination factors

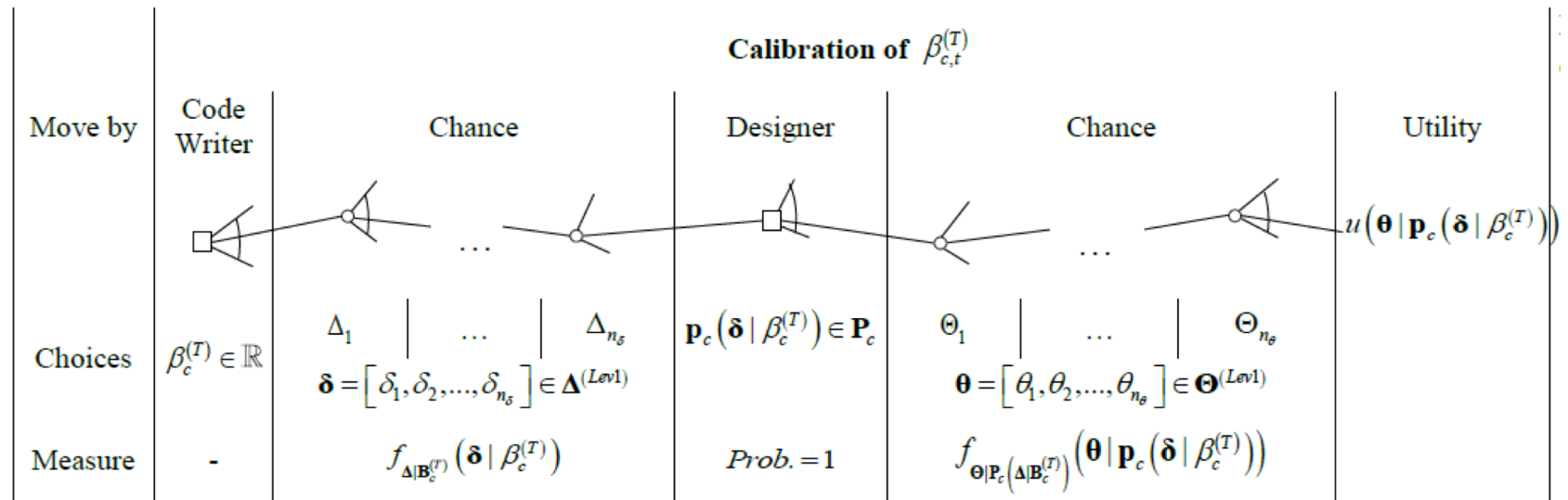


e.g. type of structure

# Simplified decision problem

## 1. Optimise $\beta_{c,target}$

- Decision variables:  $\beta_{c,target}$  for Level 1 design for a class of structures



## 2. Reliability-based calibration

- $\mathbf{r}_{el,opt}: \beta_c(\mathbf{r}_{el})$  as close as possible to  $\beta_{c,target} = \beta_{c,opt}$

# Code Calibration Overview

